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Algorithms for computation of fundamental properties of seawater, based on the practicality salinity scale (PSS-78) and the international equation of state for seawater (EOS-80), are compiled in the present report for implementing and standardizing computer programs for oceanographic data processing. Sample FORTRAN subprograms and tables are given to illustrate usage of the algorithms and to show the range of variation and limits of validity of commonly used seawater properties within the oceanic ranges for salinity, temperature, and pressure. The algorithms cover the following: conductivity to salinity conversion; salinity to conductivity conversion; specific volume anomaly and density anomaly of seawater; pressure to depth conversion; freezing point temperature of seawater; specific heat of seawater; adiabatic lapse rate; potential temperature; and sound speed in seawater. (Author)

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Algorithms for computation of fundamental properties of seawater

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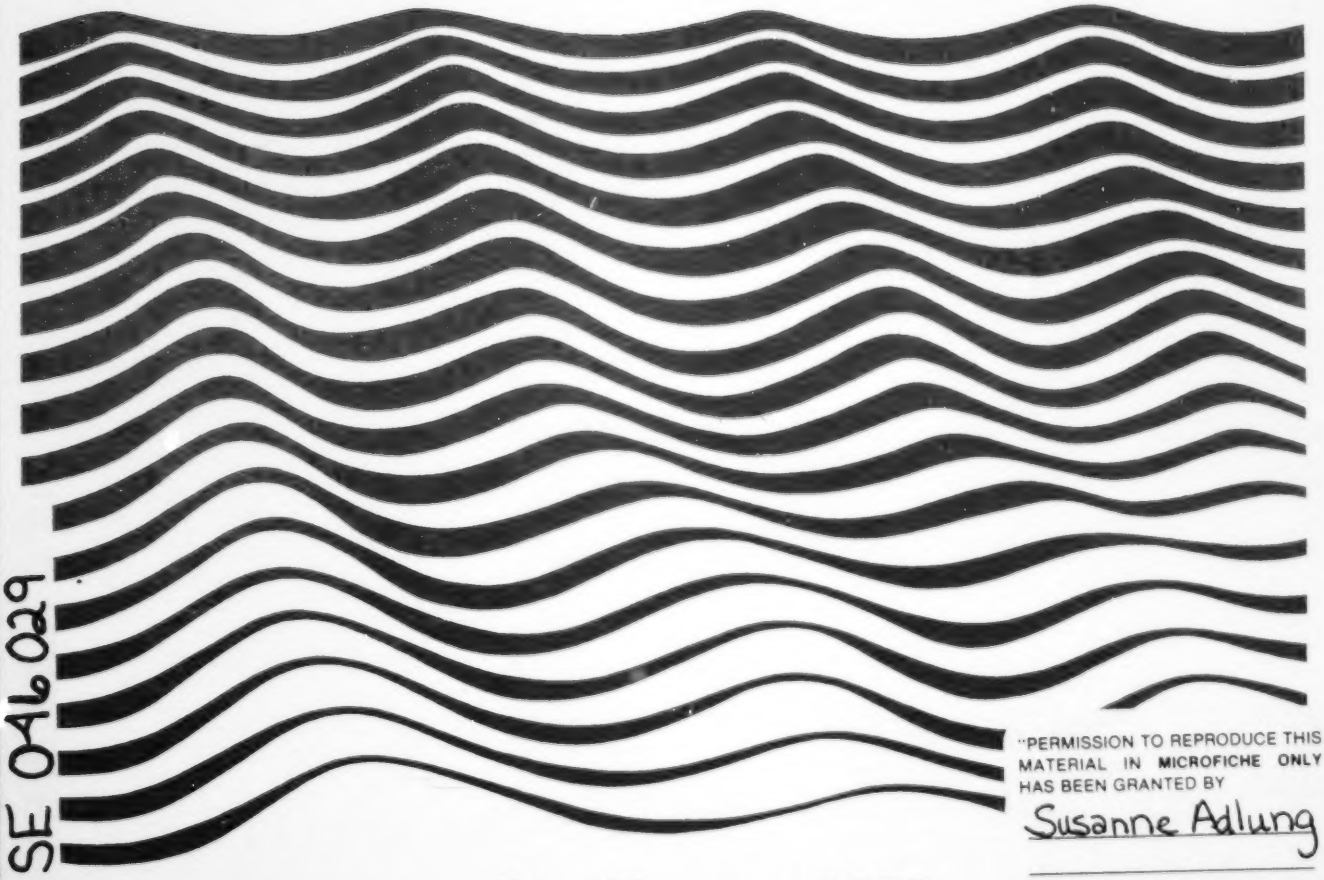

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11	An intercomparison of some current meters, report on an experiment at WHOI Mooring Site "D", 16-24 July 1967 by the Working Group on Continuous Current Velocity Measurements. Sponsored by SCOR, IAPSO and Unesco	1969	WG 21
15	Monitoring life in the ocean; sponsored by SCOR, ACMRR, Unesco, IBP/PM	1973	WG 29
16	Sixth report of the joint panel on oceanographic tables and standards, Kiel, 24-26 January 1973; sponsored by Unesco, ICES, SCOR, IAPSO	1974	WG10
19	Marine Science Teaching at the University Level. Report of the Unesco Workshop on University Curricula-*Available in French, Spanish and Arabic*	1974	—
24	Seventh report of the joint panel on oceanographic tables and standards, Grenoble, 2-5 September 1975; sponsored by Unesco, ICES, SCOR, IAPSO	1976	WG 10
25	Marine science programme for the Red Sea: Recommendations of the workshop held in Bremerhaven, FRG, 22-23 October 1974; sponsored by the Deutsche Forschungsgemeinschaft and Unesco	1976	—
26	Marine science in the Gulf area-Report of a consultative meeting, Paris, 11-14 November 1975	1976	—
27	Collected reports of the joint panel on oceanographic tables and standards, 1964-1969	1976	WG 10
28	Eighth report of the joint panel on oceanographic tables and standards, Woods Hole, U.S.A., sponsored by Unesco, ICES, SCOR, IAPSO	1978	WG 10
29	Committee for the preparation of CLOFETA-Report of the first meeting, Paris, 16-18 January 1978	1979	—
30	Ninth report of the joint panel on oceanographic tables and standards, Unesco, Paris, 11-13 September 1978	1979	—
31	Coastal lagoon survey (1976-1978)	1980	—
32	Coastal lagoon research, present and future, Report and guidelines of a seminar, Duke University Marine Laboratory, Beaufort, NC, U.S.A. August 1978. (Unesco, IABO).	1981	—
33	Coastal lagoon research, present and future. Proceedings of a seminar, Duke University, August 1978, (Unesco, IABO).	1981	—
34	The carbon budget of the oceans. Report of a meeting, Paris, 12-13 November 1979	1980	WG 62
35	Determination of chlorophyll in seawater. Report of intercalibration tests sponsored by SCOR and carried out by C.J. Lorenzen and S.W. Jeffrey, CSIRO Cronulla, N.S.W., Australia, September-October 1978	1980	—
36	The practical salinity scale 1978 and the international equation of state of seawater 1980. Tenth report of the Joint Panel on Oceanographic Tables and Standards, Sidney, B.C., Canada, 1-5 September 1980. Sponsored by Unesco, ICES, SCOR, IAPSO.	1981	WG 10
37	Background papers and supporting data on the Pratical Salinity Scale 1978.	1981	WG 10
38	Background papers and supporting data on the International Equation of State of Seawater 1980.	1981	WH 10
39	International Oceanographic Tables, Vol. 3	1981	WG 10
40	International Oceanographic Tables, Vol. 4. (To be published)	1982	WG 10
41	Ocean-Atmosphere Materials exchange (OAMEX) Report of SCOR Working Group 44, Unesco, Paris, 14-16 November 1979	1982	WG 44
42	Carbon dioxide sub-group of the joint panel on oceanographic tables and standards. Report of a meeting Miami, Florida, 21-23 September 1981 sponsored by Unesco, ICES, SCOR, IAPSO	1983	—
43	International Symposium on Coastal lagoons Bordeaux, France, 8-14 September 1981	1982	—

PREFACE

This series, the Unesco Technical Papers in Marine Science, is produced by the Unesco Division of Marine Sciences as a means of informing the scientific community of recent developments in oceanographic research and marine science affairs.

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Abstract

Algorithms for computation of fundamental properties of seawater, based on the practical salinity scale (PSS-78) and the international equation of state for seawater (EOS-80), are compiled in the present report for implementing and standardizing computer programs for oceanographic data processing.

Sample FORTRAN subprograms and tables are given to illustrate usage of the algorithms and to show the range of variation and limits of validity of commonly used seawater properties within the oceanic ranges for salinity, temperature and pressure. The algorithms cover the following: conductivity to salinity conversion; salinity to conductivity conversion; specific volume anomaly and density anomaly of seawater; pressure to depth conversion; freezing point temperature of seawater; specific heat of seawater; adiabatic lapse rate; potential temperature; sound speed in seawater.

Résumé

Des algorithmes pour le calcul des propriétés fondamentales de l'eau de mer, fondés sur l'échelle de salinité pratique (PSS-78) et l'équation d'état internationale de l'eau de mer (EOS-80), sont réunis dans le présent rapport qui a pour objectif la mise en oeuvre et la standardisation de programmes informatiques destinés au traitement des données océanographiques.

Un choix de sous-programmes en FORTRAN et de tables illustrent l'utilisation des algorithmes; les plages de variation des propriétés de l'eau de mer couramment utilisées correspondant aux gammes de valeur que prennent dans l'océan la salinité, la température et la pression sont mises en évidence, de même que les limites de validité des algorithmes. Ceux-ci couvrent les points suivants: conversion de la conductivité en salinité; conversion de la salinité en conductivité; anomalies de volume spécifique et de densité de l'eau de mer; conversion de la pression en immersion; température du point de congélation de l'eau de mer; gradient vertical de température adiabatique; température potentielle; vitesse du son dans l'eau de mer.

Resumen

Este informe, vinculado a la implementación y estandarización de programas de computación para el procesamiento de datos oceanográficos, presenta algoritmos para calcular propiedades básicas del agua de mar basados en la escala práctica de salinidad (PSS-78) y la ecuación internacional del estado del agua de mar (EOS-80).

Se presentan ejemplos de sub-programas FORTRAN y de tablas para ilustrar el uso de los algoritmos y mostrar el rango de variación y limites de validez de propiedades del agua de mar de uso común dentro de los rangos oceánicos de la salinidad, temperatura y presión. Los algoritmos cubren: conversión de conductividad en salinidad; conversión de salinidad en conductividad; anomalia del volúmen específico y anomalia de la densidad del agua de mar; conversión de la presión en profundidad; temperatura de congelamiento para el agua de mar; calor específico para el agua de mar; tasa del lapso adiabático; temperatura potencial; velocidad de sonido en el agua de mar.

Аннотация

Настоящий отчет содержит алгоритмы для вычисления некоторых свойств водных масс, основанные на практической шкале солености (PSS-78) и международном уравнении состояния воды (EOS-80). Работа выполнена с целью разработки и стандартизации программ для ЭВМ, предназначенных для обработки океанографических данных.

Приведены некоторые подпрограммы на языке Фортран и таблицы, которые иллюстрируют использование алгоритмов и показывают диапазон колебаний и пределы точности обычно измеряемых физических параметров морской воды - солености, температуры и давления в пределах их возможных значений в Мировом океане. Алгоритмы составлены для следующих процедур: перевода электропроводности в значения солености и солености - в значения электропроводности, вычисления значений конкретных аномалий объема и плотности воды, перевод величины давления в глубину, вычисления точки замерзания морской воды, теплосодержания, скорости адиабатических изменений, потенциальной температуры, а также скорости распространения звука в морской воде.

" ملخص "

يتضمن التقرير الحالى الحسابيات (الغوريثمات) الخاصة بحساب قيم الخواص الأساسية لماء البحر ، بناء على مقياس الملوحة العملية (١٩٧٨) والمعادلة العالمية لحالة ماء البحر (١٩٨٠) وذلك لتوحيد برامج الحاسب الألكترونى واستخدامها فى معالجة البيانات الأوقيانوغرافية .

وقد قدمت عينات من برامج فرعية معدّة بلغة " الفورتران " مع جداول لتوضيح استعمال هذه الحسابيات ولتبيان مدى التغير وحدود صحة انطباق الخواص الشائعة الاستعمال لمياه البحر فى ظل معدلات الملوحة والحرارة والضغط فى المحيطات .

وتغطى الحسابيات الواردة بالتقرير العمليات الآتية : تحويل درجة التوصيل الكهربائى الى الملوحة ودرجة الملوحة الى التوصيل الكهربائى ، حساب الجيود فى كل من الحجم النوعى وكثافة ماء البحر ، تحويل الضغط الى عمق ، تعيين كل من درجة حرارة التجمد والحرارة النوعية لماء البحر ، معدل التغير " الأدياباتيكى " للحرارة ، حساب درجة حرارة الوضع وسرعة الصوت فى ماء البحر .

摘　　要

本报告列举了以实用盐度表（PSS—78）和国际海水物态方程（EOS 80）为基础的海水基本特性计算法，以便实行标准化的计算机程序，供海洋学数据处理使用。

提供了FORTRAN语言的子程序和图表的实例来说明计算法的使用，并表明通常使用的海水特性在海洋盐度、温度和压力的范围之内的变化范围与有效限度。这些计算法包括以下各个方面：传导率—— 盐度转换；盐度—— 传导率转换；海水的比容偏差和密度偏差；深度转换压力；海水的冰点温度；海水的比热；绝热递减率；势温；海水中声速。

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Foreword

The algorithms compiled in the present report are based on the practical salinity scale and the international equation of state for seawater endorsed by UNESCO/SCOR/ICES/IAPSO Joint Panel on Oceanographic Tables and Standards and SCOR Working Group 51 on evaluation of CTD data.

Compilation of the algorithms and tables was supported by the Office of Naval Research under contracts N00014-74-C-0262, NR033-004 and N00014-76-C-0197, NR083-400 and by the National Science Foundation under Grant OCE 78-06886.00.

Introduction

The recent adoption for general oceanographic use of the practical salinity scale (PSS-78) and the international equation of state for seawater (EOS80) by the Joint Panel on Oceanographic Tables and Standards (Unesco Report No. 38, 1981) has created a need for a consistent set of algorithms for computing various derived properties from the basic observations of electrical conductivity ratio, temperature and pressure. Formulas based on earlier salinity and temperature scales are with few exceptions not recommended for use with the new practical salinities.

Lewis and Perkin (1981) examined differences between PSS-78 salinities and salinities obtained from several formulas developed for the previous salinity scale. Within the oceanic range, the differences are less than 0.01 but larger than the precision of measurement (± 0.003). For more diluted or concentrated seawaters, the differences are as high as 0.06 (Lewis and Perkin, 1981, Tables 2-7). Temperatures based on the International Practical Temperature Scale (IPTS-68) differ from the 1948 scale (IPTS-48) by as much as .009 °C (Barber, 1969) over the oceanic range (-2 to 40°C). Because both temperature and salinity scales differences exceed the precision of measurement of modern sensors, corrections or conversions are necessary to maintain precision in derived variables for temperature and salinity data obtained on the previous scales.

The 1980 equation of state (EOS80) provides a significant improvement in accuracy of density and specific volume of seawater over the entire oceanic range. Thermal expansion, compressibility, specific heat, adiabatic lapse rate, potential temperature and sound speed can be computed with improved accuracy from EOS80.

A selection of algorithms, consistent with practical salinity (PSS-78) and temperature (IPTS-68) are compiled in the present report. Formulas documented in publications were compared with direct calculations using EOS80 to establish consistency.

Formulas are given for:

1. Conductivity ratio to practical salinity conversion (SAL78)
2. Practical salinity to conductivity ratio conversion (SAL78)
3. Specific volume anomaly SVAN(S,t,p,SIGMA); Density anomaly SIGMA
4. Pressure to depth conversion DEPTH (p,latitude)
5. Freezing point temperature for seawater TF(S,P)
6. Specific heat of seawater CPSW(S,t,p)
7. Adiabatic lapse rate ATG(S,t,p)
8. Potential temperature THETA(S,t,p,p_r)
9. Sound speed in seawater SVEL(S,t,p)

A sample FORTRAN subprogram is included for each formula together with a test value, error estimates and a summary table of typical values in the oceanographic range. Summary tables extending beyond the range of the algorithms, have a border drawn around values within the range of

validity (see Salinity Table Page 8). The call list input variables are S=salinity (PSS-78); t=temperature in degrees Celsius (IPTS-1968); p=pr= pressure in decibars (1 decibar = 10^4 Pa). It should be noted that the pressure coefficients of some of the subprograms have units of bars (Formulas 3,5, and 8 in list above). A rescaling of input pressure from decibars to bars is found in these subprograms. Depth requires latitude in degrees and the specific volume anomaly function returns density anomaly in the call list. The check values are computed for a 32-bit word length with a 24-bit mantissa (using an implicit leading bit) yielding a numerical resolution of 2^{-24} or 6×10^{-8}.

1. Conductivity Ratio to Salinity Conversion

If C(S,t,p) is electrical conductivity of seawater at salinity S (PSS-78), temperature t (IPTS-68) and pressure p decibars, the conductivity ratio is defined to be

$$R = C(S,t,p)/C(35,15,0)$$

where C[35,15,0] is the conductivity of standard seawater of practical salinity 35, at 15°C and atmospheric pressure, defined to be equal to the conductivity of a reference solution of potassium chloride (KCl) at the same temperature and pressure. This KCl reference contains 32.4356 grams of KCl in a mass of one kilogram of solution.

The conductivity ratio can be factored into three parts, i.e.,

$$R = R_p \cdot R_t \cdot r_t$$

where

$$R_p(S,t,p) = C(S,t,p)/C(S,t,0)$$

$$R_t(S,t) = C(S,t,0)/C(35,t,0)$$

$$r_t(t) = C(35,t,0)/C(35,15,0)$$

As the ratios R_t or R are measured directly with conductivity bridges together with temperature and pressure, formulas have been developed for the ratios in these variables rather than in salinity S.

The practical salinity, therefore, may be computed with the equation

$$S = a_0 + a_1 R_t^{1/2} + a_2 R_t + a_3 R_t^{3/2} + a_4 R_t^2 + a_5 R_t^{5/2} + \Delta S \qquad (1)$$

where

$$\Delta S = \frac{(t-15)}{1+k(t-15)} \left(b_0 + b_1 R_t^{1/2} + b_2 R_t + b_3 R_t^{3/2} + b_4 R_t^2 + b_5 R_t^{5/2}\right) \qquad (2)$$

with the constants a_i defined in Equation (1) and

a_0	= + 0.0080	b_0	= + 0.0005	
a_1	= - 0.1692	b_1	= - 0.0056	
a_2	= + 25.3851	b_2	= - 0.0066	
a_3	= + 14.0941	b_3	= - 0.0375	k = + 0.0162
a_4	= - 7.0261	b_4	= + 0.0636	
a_5	= + 2.7081	b_5	= - 0.0144	
Σa_i	= 35.0000	Σb_i	= 0.0000	

. $2 < S < 42$

The equations (1) and (2) are valid over the ranges of temperature (-2 to 35°C), and practical salinity (2-42) in the world ocean (Perkin and Lewis, 1980).

The ratio r_t is given by

$$r_t = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 \qquad (3)$$

where

c_0 = + 0.6766097

c_1 = + 2.00564 E-2

c_2 = + 1.104259 E-4

$$c_3 = -6.9698 \quad E\text{-}7$$

$$c_4 = +1.0031 \quad E\text{-}9$$

Range of validity: $t = -2$ to $35^\circ C$

The ratio R_p is given by

$$R_p = 1 + \frac{p(e_1 + e_2 p + e_3 p^2)}{1 + d_1 t + d_2 t^2 + (d_3 + d_4 t)R} \qquad (4)$$

where

$e_1 = +2.070 \quad E\text{-}5$ $\qquad$ $d_1 = +3.426 \quad E\text{-}2$

$e_2 = -6.370 \quad E\text{-}10$ $\qquad$ $d_2 = +4.464 \quad E\text{-}4$

$e_3 = +3.989 \quad E\text{-}15$ $\qquad$ $d_3 = +4.215 \quad E\text{-}1$

$\qquad\qquad\qquad\qquad\quad$ $d_4 = -3.107 \quad E\text{-}3$

Maxima of the range of pressures (decibars) over which the pressure correction to conductivity ratio was computed is given in the following table (Perkin and Lewis, 1980);

T°C/S	T < 0	5	10	15	20	25	30
2							
14	1998	1998	1998	1998	1998	1998	1998
22	1998	1998	3996	1998	1998	1998	1998
31	1722	1722	1722	1722	1722	1722	
35	10,334	9988	3996	1998	1998	1998	1998
39	1722	1722	5162	5162	1722	1722	

Given R,t,p, the ratio R_t is given by

$$R_t = R/(R_p \cdot r_t)$$

and salinity is computed from (1) and (2).

The following values are provided for checking the correct use of the above equations.

R	t (°C)	p (dbars)	R_p	r_t	R_t	S
1	15	0	1.0000000	1.0000000	1.0000000	35.000000
1.2	20	2000	1.0169429	1.1164927	1.0568875	37.245628
0.65	5	1500	1.0204864	0.77956585	0.81705885	27.995347

2. Salinity to Conductivity Ratio Conversion

If the conductivity ratio is required, the salinity algorithm can be inverted numerically by an iterative computation.

Given salinity S and temperature t, the conductivity ratio R_t is found by inverting formulas (1) and (2), i.e.,

$$S = S(R_t, t) \quad . \tag{5}$$

If a value R_{tn} is known, the corresponding value of S_n is found from (5). As S_n will differ from S, a better approximation R_{tn+1} is obtained from the Taylor expansion of (5),

$$S - S_n = \frac{\partial S}{\partial R_t} (R_{tn+1} - R_{tn})$$

i.e.,

$$R_{tn+1} = R_{tn} + (S - S_n)/\partial S/\partial R_t \ .$$

This procedure is the standard Newton-Raphson iteration if the derivative $\partial S/\partial R_t$ is evaluated directly by differentiating the formula for S. In the actual computation it is more convenient to differentiate with respect to $\sqrt{R}_t$ and solve for $\sqrt{R}_t$ because of the half powers in the formula.

If R_t is known, the value of R is computed by solving the quadratic equation obtained from

$$R = r_t \cdot R_t \cdot R_p = r_t \cdot R_t \cdot \left(1 + \frac{C}{AR + B}\right) \tag{6}$$

$$R = \frac{1}{2A}\left[\mathrm{SQRT}[(B - Ar_tR_t)^2 + 4r_tR_tA(B + C)] - (B - Ar_tR_t)\right]$$

where A, B and C are polynomials in temperature and pressure given in (4).

Both the conductivity ratio to salinity and salinity to conductivity ratio formulas are combined in a single function subprogram. Although the Practical Salinity Scale is not defined below 2, the subroutine returns non-zero values above 0.02. For conductivity ratios of less than 0.0005 or salinities less than 0.02, the function is forced to return a zero value to avoid convergence problems for zero input values of salinity or conductivity.

```
      REAL FUNCTION SAL78(CND,T,P,M)
C ***************************************************************************
C     THE CONDUCTIVITY RATIO (CND) = 1.0000000 FOR SALINITY = 35 PSS-78
C     TEMPERATURE = 15.0 DEG. CELSIUS , AND ATMOSPHERIC PRESSURE.
C***************************************************************************
C
C FUNCTION TO CONVERT CONDUCTIVITY RATIO TO SALINITY (M = 0)
C SALINITY TO CONDUCTIVITY RATIO (M = 1,CND BECOMES INPUT SALINITY)
C***************************************************************************
C   REFERENCES:   ALSO LOCATED IN UNESCO REPORT NO. 37 1981
C  PRACTICAL SALINITY SCALE 1978: E.L. LEWIS IEEE OCEAN ENG. JAN. 1980
C ***************************************************************************
C UNITS:
C       PRESSURE         P        DECIBARS
C       TEMPERATURE      T        DEG CELSIUS (IPTS-68)
C       CONDUCTIVITY     CND      RATIO      (M=0)
C       SALINITY         SAL78    (PSS-78)  (M=0)
C  CHECKVALUES:
C      SAL78=1.888091 :CND= 40.0000,T=40 DEG C,P= 10000 DECIBARS:   M= 1
C      SAL78=40.00000 :CND=1.888091,T=40 DEG C,P=10000 DECIBARS:    M=0
C***************************************************************************
C SAL78 RATIO: RETURNS ZERO FOR CONDUCTIVITY RATIO:  < 0.0005
C SAL78: RETURNS ZERO FOR SALINITY:  < 0.02
C ***************************************************************************
C INTERNAL FUNCTIONS
C ***************************************************************************
C  PRACTICAL SALINITY SCALE 1978 DEFINITION WITH TEMPERATURE CORRECTION
C  XT=T-15.0 : XR=SQRT(RT)
      SAL(XR,XT) =(((2.7081*XR-7.0261)*XR+14.0941)*XR+25.3851)*XR
     X-0.1692)*  XR+0.0080
     X  +(XT/(1.0+0.0162*XT))*(((((-0.0144*XR+
     X   0.0636)*XR-0.0375)*XR-0.0066)*XR-0.0056)*XR+0.0005)
C  DSAL(XR,XT)  FUNCTION FOR DERIVATIVE OF SAL(XR,XT) WITH XR.
      DSAL(XR,XT) =((((13.5405*XR-28.1044)*XR+42.2823)*XR+50.7702)*XR
     X   -0.1692)+(XT/(1.0+0.0162*XT))*((((-0.0720*XR+0.2544)*XR
     X   -0.1125)*XR-0.0132)*XR-0.0056)
C  FUNCTION RT35 :  C(35,T,0)/C(35,15,0) VARIATION WITH TEMPERATURE
C  WITH TEMPERATURE.
      RT35(XT) = (((1.0031E-9*XT-6.9698E-7)*XT+1.104259E-4)*XT
     X   + 2.00564E-2)*XT + 0.6766097
C POLYNOMIALS OF RP: C(S,T,P)/C(S,T,0) VARIATION WITH PRESSURE
C  C(XP) POLYNOMIAL CORRESPONDS TO A1-A3 CONSTANTS: LEWIS 1980
      C(XP) = ((3.989E-15*XP-6.370E-10)*XP+2.070E-5)*XP
      B(XT) = (4.464E-4*XT+3.426E-2)*XT + 1.0
C  A(XT) POLYNOMIAL CORRESPONDS TO B3 AND B4 CONSTANTS: LEWIS 1980
      A(XT) = -3.107E-3*XT + 0.4215
C***************************************************************************
```

```
C ZERO SALINITY/CONDUCTIVITY TRAP
      SAL78=0.0
      IF((M.EQ.0).AND.(CND.LE.5E-4)) RETURN
      IF((M.EQ.1).AND.(CND.LE.0.02)) RETURN
C ***********************************************
      DT = T - 15.0
C SELECT BRANCH FOR SALINITY (M=0) OR CONDUCTIVITY (M=1)
      IF(M.EQ.1)  GO TO 10
C ***********************************************
C CONVERT CONDUCTIVITY TO SALINITY
      R = CND
      RT = R/(RT35(T)*(1.0 + C(P)/(B(T) + A(T)*R)))
      RT = SQRT(ABS(RT))
      SAL78 = SAL(RT,DT)
      RETURN
C *********  END OF CONDUCTIVITY TO SALINITY SECTION **********
C ********************************************************
C INVERT SALINITY TO CONDUCTIVITY BY THE
C  NEWTON-RAPHSON ITERATIVE METHOD.
C ************************************************************
C FIRST APPROXIMATION
   10 RT = SQRT(CND/35.0)
      SI = SAL(RT,DT)
      N = 0
C
C  ITERATION LOOP BEGINS HERE WITH A MAXIMUM OF 10 CYCLES
C
   15 RT = RT + (CND - SI)/DSAL(RT,DT)
      SI = SAL(RT,DT)
      N = N + 1
      DELS = ABS(SI - CND)
      IF((DELS.GT.1.0E-4).AND.(N.LT.10))GO TO 15
C
C ******************************END OF ITERATION LOOP ********
C
C COMPUTE CONDUCTIVITY RATIO
      RTT = RT35(T)*RT*RT
      AT = A(T)
      BT = B(T)
      CP = C(P)
      CP = RTT*(CP + BT)
      BT = BT - RTT*AT
C
C SOLVE QUADRATIC EQUATION FOR R: R=RT35*RT*(1+C/AR+B)
C
      R = SQRT(ABS(BT*BT + 4.0*AT*CP)) - BT
C CONDUCTIVITY RETURN
      SAL78 = 0.5*R/AT
      RETURN
      END
```

SALINITY S [PSS-78]

PRESSURE DECIBARS	TEMPERATURE °C IPTS-68 0	10	20	30	40	COND. RATIO : 0.6990725
0	36.2864	26.8609	20.8085	16.7100	13.8131	
1000	35.6746	26.5072	20.5932	16.5725	13.7213	
2000	35.1188	26.1836	20.3953	16.4456	13.6364	
3000	34.6143	25.8880	20.2136	16.3286	13.5579	
4000	34.1568	25.6183	20.0470	16.2211	13.4855	
5000	33.7426	25.3729	19.8948	16.1225	13.4191	
6000	33.3684	25.1501	19.7562	16.0324	13.3582	
7000	33.0314	24.9485	19.6303	15.9505	13.3027	
8000	32.7289	24.7669	19.5166	15.8762	13.2523	
9000	32.4588	24.6042	19.4143	15.8093	13.2069	
10000	32.2191	24.4592	19.3231	15.7495	13.1662	
						COND. RATIO :
0	50.0127	36.9676	28.6080	22.9533	18.9594	0.9320967
1000	49.2140	36.4999	28.3203	22.7681	18.8351	
2000	48.4874	36.0717	28.0556	22.5970	18.7200	
3000	47.8269	35.6801	27.8125	22.4394	18.6135	
4000	47.2272	35.3226	27.5896	22.2943	18.5154	
5000	46.6837	34.9970	27.3858	22.1614	18.4253	
6000	46.1921	34.7013	27.2001	22.0399	18.3427	
7000	45.7489	34.4336	27.0314	21.9292	18.2674	
8000	45.3509	34.1923	26.8790	21.8290	18.1991	
9000	44.9952	33.9760	26.7419	21.7388	18.1375	
10000	44.6791	33.7832	26.6195	21.6580	18.0822	
						COND. RATIO :
0	64.4641	47.5215	36.7173	29.4273	24.2862	1.1651209
1000	63.4810	46.9414	36.3576	29.1942	24.1289	
2000	62.5855	46.4098	36.0265	28.9788	23.9832	
3000	61.7706	45.9233	35.7222	28.7802	23.8485	
4000	61.0299	45.4790	35.4432	28.5975	23.7244	
5000	60.3579	45.0741	35.1879	28.4300	23.6102	
6000	59.7498	44.7060	34.9552	28.2769	23.5057	
7000	59.2011	44.3727	34.7439	28.1375	23.4104	
8000	58.7080	44.0722	34.5527	28.0111	23.3239	
9000	58.2670	43.8026	34.3809	27.8973	23.2458	
10000	57.8750	43.5623	34.2274	27.7955	23.1759	
						COND. RATIO :
0	79.7047	58.5321	45.1289	36.1195	29.7800	1.3981451
1000	78.5355	57.8397	44.6972	35.8382	29.5894	
2000	77.4695	57.2050	44.2997	35.5783	29.4128	
3000	76.4986	56.6237	43.9342	35.3386	29.2496	
4000	75.6154	56.0925	43.5989	35.1181	29.0991	
5000	74.8136	55.6082	43.2922	34.9158	28.9607	
6000	74.0874	55.1678	43.0124	34.7308	28.8340	
7000	73.4318	54.7688	42.7583	34.5624	28.7184	
8000	72.8423	54.4088	42.5284	34.4098	28.6135	
9000	72.3148	54.0858	42.3216	34.2723	28.5188	
10000	71.8457	53.7979	42.1369	34.1492	28.4340	

CONDUCTIVITY RATIO R

PRESSURE DECIBARS	TEMPERATURE °C IPTS-68 0	10	20	30	40	SALINITY: 25
0	0.498008	0.654990	0.825481	1.006639	1.195796	
1000	0.506244	0.662975	0.833139	1.013951	1.202767	
2000	0.513925	0.670435	0.840302	1.020797	1.209296	
3000	0.521068	0.677385	0.846983	1.027187	1.215394	
4000	0.527692	0.683840	0.853194	1.033130	1.221069	
5000	0.533813	0.689814	0.858947	1.038640	1.226330	
6000	0.539447	0.695319	0.864254	1.043724	1.231189	
7000	0.544609	0.700369	0.869126	1.048395	1.235653	
8000	0.549314	0.704976	0.873574	1.052661	1.239732	
9000	0.553574	0.709153	0.877610	1.056533	1.243435	
10000	0.557404	0.712912	0.881243	1.060020	1.246770	
0	0.588220	0.772567	0.972727	1.185361	1.407338	SALINITY: 30
1000	0.597649	0.781728	0.981533	1.193791	1.415394	
2000	0.606437	0.790282	0.989769	1.201681	1.422939	
3000	0.614605	0.798249	0.997447	1.209045	1.429984	
4000	0.622176	0.805646	1.004585	1.215893	1.436539	
5000	0.629168	0.812488	1.011194	1.222240	1.442617	
6000	0.635602	0.818792	1.017289	1.228097	1.448228	
7000	0.641494	0.824574	1.022884	1.233476	1.453382	
8000	0.646862	0.829847	1.027990	1.238388	1.458092	
9000	0.651722	0.834627	1.032622	1.242846	1.462367	
10000	0.656090	0.838926	1.036792	1.246860	1.466219	
0	0.676610	0.887529	1.116493	1.359679	1.613508	SALINITY: 35
1000	0.687138	0.897778	1.126369	1.369156	1.622586	
2000	0.696945	0.907346	1.135602	1.378024	1.631085	
3000	0.706057	0.916254	1.144210	1.386298	1.639020	
4000	0.714499	0.924521	1.152207	1.393993	1.646403	
5000	0.722293	0.932167	1.159613	1.401122	1.653247	
6000	0.729461	0.939209	1.166440	1.407701	1.659565	
7000	0.736025	0.945666	1.172706	1.413741	1.665369	
8000	0.742002	0.951555	1.178425	1.419257	1.670672	
9000	0.747413	0.956891	1.183612	1.424263	1.675484	
10000	0.752275	0.961690	1.188279	1.428769	1.679819	
0	0.763298	1.000073	1.257061	1.529967	1.814775	SALINITY: 40
1000	0.774845	1.011334	1.267926	1.540427	1.824816	
2000	0.785595	1.021844	1.278102	1.550212	1.834216	
3000	0.795580	1.031625	1.287575	1.559342	1.842990	
4000	0.804827	1.040701	1.296377	1.567830	1.851154	
5000	0.813362	1.049093	1.304525	1.575694	1.858721	
6000	0.821209	1.056821	1.312037	1.582949	1.865705	
7000	0.828392	1.063905	1.318929	1.589610	1.872121	
8000	0.834932	1.070363	1.325218	1.595692	1.877982	
9000	0.840851	1.076215	1.330921	1.601210	1.883301	
10000	0.846168	1.081478	1.336054	1.606179	1.888091	

3. Specific Volume Anomaly and Density Anomaly of Seawater

The international equation of state for seawater diluted with pure water or concentrated by evaporation has been determined to be used with the practical salinity scale (Millero et al., 1980; Millero and Poisson, 1981). This equation is more precise than the currently used equations (Knudsen et al., 1902; Ekman, 1908; Cox et al., 1970) and covers a wider range of temperature and pressure. Data reports describing the details of the fitting procedure are available (Millero, Chen, Bradshaw and Schleicher, Unesco Report No. 38 (1981); Millero and Poisson, Unesco Report No. 38 (1981).

The density (ρ, kg/m^3) of seawater as a function of practical salinity (S), temperature (t, °C) and applied or gauge pressure (p, decibars) is given by

$$\rho(S,t,p) = \rho(S,t,0)/[1-p/K(S,t,p)] \qquad (7)$$

where K(S,t,p) is the secant bulk modulus. The specific volume ($V = 1/\rho$ m^3/kg) of seawater can be obtained from

$$V(S,t,p) = V(S,t,0)\cdot[1-p/K(S,t,p)] \qquad (8)$$

Because the maximum variation of density and specific volume of sea water is less than 7 percent over the oceanic range of variables, numerical precision is lost by retaining the full numerical magnitude. A 32 bit single precision floating point number has about 7 significant decimal digits. While such precision exceeds the accuracy of the measurements and is adequate for most purposes where the full value is required, it is insufficient for taking differences for Brunt-Vaisala frequency calculations. An improvement by two orders of magnitude in the oceanographic range is obtained by computing specific volume anomaly δ defined by:

$$\delta = V(S,t,p) - V(35,0,p) \qquad (9)$$

and reported in units of 10^{-8} m^3/kg.

For many oceanographic applications, the specific volume (or steric) anomaly δ provides a more convenient measure of the baroclinic structure in the ocean. For computational convenience, V(35,0,p) has been obtained from the EOS80 by evaluating the coefficients for a salinity of 35 (PSS-78) and temperature of 0°C (IPTS-68) in the form

$$V(35,0,p) = V(35,0,0)\cdot[1 - p/K(35,0,p)] \qquad (10)$$

$$K(35,0,p) = K_o + A_o p + B_o p^2$$

where

$$V(35,0,0) = 9.7266204\ E\text{-}4\ \ m^3/kg$$

$$K_o = +\ 21582.27$$

$$A_o = +\ 3.35940552$$

$$B_o = +\ 5.03217\ \ E\text{-}5$$

To achieve the increased precision, terms in the difference equation for δ that are functions of pressure only must be eliminated. The pressure terms can be subtracted by reformulating the equation for δ in terms of the difference $K(S,t,p)-K(35,0,p)$ as follows:

$$\delta = V(S,t,0)\cdot\left[1 - \frac{p}{K(S,t,p)} + \frac{p}{K(35,0,p)} - \frac{p}{K(35,0,p)}\right]$$

$$-V(35,0,0)\cdot\left|1 - \frac{p}{K(35,0,p)}\right|$$

$$= \delta(S,t,0)\cdot\left[1 - \frac{p}{K(35,0,p)}\right] + V(S,t,0)\cdot\left[\frac{p}{K(35,0,p)} - \frac{p}{K(S,t,p)}\right]$$

$$\delta = \delta(S,t,0)\cdot\left[1 - \frac{p}{K(35,0,p)}\right] + V(S,t,0)\cdot p\cdot\left[\frac{K(S,t,p) - K(35,0,p)}{K(S,t,p)\cdot K(35,0,p)}\right] \quad (11)$$

Improvement in the numerical precision is achieved by subtracting the coefficients in the formulas for $\delta(S,T,0)$ and $K(S,T,p)-K(35,0,p)$ prior to coding (see terms marked with asterisks).

The density anomaly σ is defined by

$$\sigma = \frac{1}{V(S,t,p)} - 1000\ kg/m^3$$

The formula for σ, obtained by a similar derivation, is

$$\sigma = \rho(35,0,0) - 1000\ kg/m^3 + \frac{p}{V(35,0,p)K(35,0,p)} - \frac{\delta}{V(35,0,p)V(S,t,p)} \quad (12)$$

where $\rho(35,0,0)-1000\ kg/m^3 = 28.106331\ kg/m^3$

It should be noted that this procedure improves the numerical precision only and does not effect the accuracy of the 1980 equation of state. The accuracy is determined by the laboratory measurements used to construct the equation.

The asterisked constants that follow are modified from the constants given in Millero and Poisson (1980) to permit the computation of specific volume anomaly δ. The modified constants necessary for specific volume anomaly computations are enclosed in brackets.

$$\rho(S,t,0) = \rho_w + (b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4)S + (c_0 + c_1 t + c_2 t^2)S^{3/2} + d_0 S^2 \quad (13)$$

b_0 = + 8.24493E-1 $\quad$ c_0 = - 5.72466E-3

b_1 = - 4.0899 E-3 $\quad$ c_1 = + 1.0227 E-4

b_2 = + 7.6438 E-5 $\quad$ c_2 = - 1.6546 E-6

b_3 = - 8.2467 E-7

b_4 = + 5.3875 E-9 $\quad$ d_0 = + 4.8314 E-4

The density of the reference pure water (SMOW, Craig, 1961) is given by (IUPAC, 1976)

$$\rho_w = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 \quad (14)$$

where

a_0^* = + 999.842594 [-28.263737]

a_1 = + 6.793952 E-2

a_2 = - 9.095290 E-3

a_3 = + 1.001685 E-4

a_4 = - 1.120083 E-6

a_5 = + 6.536332 E-9

Should more reliable data for pure water become available in the future, this equation can be easily modified. The secant bulk modulus (K) of seawater is given by

$$K(S,t,p) = K(S,t,0) + Ap + Bp^2 \tag{15}$$

where

$$K(S,t,0) = K_w + (f_0 + f_1 t + f_2 t^2 + f_2 t^3)S + (g_0 + g_1 t + g_2 t^2)S^{3/2} \tag{16}$$

$f_0 = + 54.6746$ $\qquad$ $g_0 = + 7.944$ E-2

$f_1 = - 0.603459$ $\qquad$ $g_1 = + 1.6483$ E-2

$f_2 = + 1.09987$ E-2 $\qquad$ $g_2 = - 5.3009$ E-4

$f_3 = - 6.1670$ E-5

$$A = A_w + (i_0 + i_1 t + i_2 t^2)S + j_0 S^{3/2} \tag{17}$$

$i_0 = + 2.2838$ E-3 $\qquad$ $j_0 = + 1.91075$E-4

$i_1 = - 1.0981$ E-5

$i_2 = - 1.6078$ E-6

$$B = B_w + (m_0 + m_1 t + m_2 t^2)S \tag{18}$$

$m_0 = - 9.9348$ E-7

$m_1 = + 2.0816$ E-8

$m_2 = + 9.1697$ E-10

The pure water terms of the secant bulk modulus are given by

$$K_w = e_0 + e_1 t + e_2 t^2 + e_3 t^3 + e_4 t^4 \tag{19}$$

$e_0^* = + 19652.21$ [-1930.06] $\qquad$ $e_2 = - 2.327105$

$e_1 = + 148.4206$ $\qquad$ $e_3 = + 1.360477$ E-2

$e_4 = - 5.155288$ E-5

$$A_w = h_0 + h_1 t + h_2 t^2 + h_3 t^3$$

h_0^* = + 3.239908 [-0.1194975]

h_1 = + 1.43713 E-3

h_2 = + 1.16092 E-4

h_3 = - 5.77905 E-7

$$B_w = k_0 + k_1 t + k_2 t^2$$

k_0^* = + 8.50935 E-5 [+ 3.47718 E-5]

k_1 = - 6.12293 E-6

k_2 = + 5.2787 E-8

The EOS80 is valid for S = 0 to 42 ; t = -2 to 40°C; p = 0 to 10000 decibars.

The following values are provided (Unesco Report No. 38 p. 191) for checking the correct use of the above equation. The density anomaly can be compared directly with $\rho(S,t,p)$ when 1000. kg/m^3 is added. Differences in the last decimal place particularly of specific volume can be expected due to round-off. (Units of ρ are kg m^{-3} and V m^3/kg.)

S	t(°C)	p(dbars)	$\rho(S,t,p)$	$V(S,t,p)10^{-3}$ m^3/kg
0	5	0	999.96675	1.000033251
		10000	1044.12802	.957736964
	25	0	997.04796	1.00296078
		10000	1037.90204	.963482064
35	5	0	1027.67547	.973069835
		10000	1069.48914	.935025857
	25	0	1023.34306	.977189409
		10000	1062.53817	.941142660

```
      REAL FUNCTION SVAN(S,T,PO,SIGMA)
C ********************************************************
C SPECIFIC VOLUME ANOMALY (STERIC ANOMALY) BASED ON 1980 EQUATION
C OF STATE FOR SEAWATER AND 1978 PRACTICAL SALINITY SCALE.
C REFERENCES
C MILLERO, ET AL (1980) DEEP-SEA RES.,27A,255-264
C MILLERO AND POISSON 1981,DEEP-SEA RES.,28A PP 625-629.
C BOTH ABOVE REFERENCES ARE ALSO FOUND IN UNESCO REPORT NO. 38 (1981)
C UNITS:
C       PRESSURE         PO       DECIBARS
C       TEMPERATURE      T        DEG CELSIUS (IPTS-68)
C       SALINITY         S        (PSS-78)
C       SPEC. VOL. ANO.  SVAN     1.0E-8 M**3/KG
C       DENSITY ANO.     SIGMA    KG/M**3
C ********************************************************
CHECK VALUE: SVAN = 981.30210 E-8  M**3/KG FOR S = 40 (PSS-78) ,
C T = 40 DEG C, PO= 10000 DECIBARS.
C CHECK VALUE: SIGMA = 59.82037  KG/M**3 FOR S = 40 (PSS-78) ,
C T = 40 DEG C, PO= 10000 DECIBARS.
      REAL P,T,S,SIG,SR,R1,R2,R3,R4
      REAL A,B,C,D,E,A1,B1,AW,BW,K,KO,KW,K35
C EQUIV
      EQUIVALENCE (E,D,B1),(BW,B,R3),(C,A1,R2)
      EQUIVALENCE (AW,A,R1),(KW,KO,K)
C DATA
      DATA R3500,R4/1028.1063,4.8314E-4/
      DATA DR350/28.106331/
C   R4 IS REFERRED TO AS  C  IN MILLERO AND POISSON 1981
C CONVERT PRESSURE TO BARS AND TAKE SQUARE ROOT SALINITY.
      P=PO/10.
      SR = SQRT(ABS(S))
C ********************************************************
C PURE WATER DENSITY AT ATMOSPHERIC PRESSURE
C   BIGG P.H.,(1967) BR. J. APPLIED PHYSICS 8 PP 521-537.
      R1 = ((((6.536332E-9*T-1.120083E-6)*T+1.001685E-4)*T
     X-9.095290E-3)*T+6.793952E-2)*T-28.263737
C SEAWATER DENSITY ATM PRESS.
C  COEFFICIENTS INVOLVING SALINITY
C  R2 = A   IN NOTATION OF MILLERO AND POISSON 1981
      R2 = (((5.3875E-9*T-8.2467E-7)*T+7.6438E-5)*T-4.0899E-3)*T
     X+8.24493E-1
C  R3 = B  IN NOTATION OF MILLERO AND POISSON 1981
      R3 = (-1.6546E-6*T+1.0227E-4)*T-5.72466E-3
C  INTERNATIONAL ONE-ATMOSPHERE EQUATION OF STATE OF SEAWATER
      SIG = (R4*S + R3*SR + R2)*S + R1
C SPECIFIC VOLUME AT ATMOSPHERIC PRESSURE
      V350P = 1.0/R3500
      SVA = -SIG*V350P/(R3500+SIG)
      SIGMA=SIG+DR350
C  SCALE SPECIFIC VOL. ANOMALY TO NORMALLY REPORTED UNITS
      SVAN=SVA*1.0E+8
      IF(P.EQ.0.0) RETURN
```

```
C ***************************************************************
C ******  NEW HIGH PRESSURE EQUATION OF STATE FOR SEAWATER ********
C ***************************************************************
C        MILLERO, ET AL , 1980 DSR 27A, PP 255-264
C              CONSTANT NOTATION FOLLOWS ARTICLE
C*****************************************************
C COMPUTE COMPRESSION TERMS
      E = (9.1697E-10*T+2.0816E-8)*T-9.9348E-7
      BW = (5.2787E-8*T-6.12293E-6)*T+3.47718E-5
      B = BW + E*S
C
      D = 1.91075E-4
      C = (-1.6078E-6*T-1.0981E-5)*T+2.2838E-3
      AW = ((-5.77905E-7*T+1.16092E-4)*T+1.43713E-3)*T
     X-0.1194975
      A = (D*SR + C)*S + AW
C
      B1 = (-5.3009E-4*T+1.6483E-2)*T+7.944E-2
      A1 = ((-6.1670E-5*T+1.09987E-2)*T-0.603459)*T+54.6746
      KW = (((-5.155288E-5*T+1.360477E-2)*T-2.327105)*T
     X+148.4206)*T-1930.06
      KO = (B1*SR + A1)*S + KW
C EVALUATE PRESSURE POLYNOMIAL
C ***********************************************
C   K EQUALS THE SECANT BULK MODULUS OF SEAWATER
C   DK=K(S,T,P)-K(35,0,P)
C  K35=K(35,0,P)
C ***********************************************
      DK = (B*P + A)*P + KO
      K35  = (5.03217E-5*P+3.359406)*P+21582.27
      GAM=P/K35
      PK = 1.0 - GAM
      SVA = SVA*PK + (V350P+SVA)*P*DK/(K35*(K35+DK))
C  SCALE SPECIFIC VOL. ANOMALY TO NORMALLY REPORTED UNITS
      SVAN=SVA*1.0E+8
      V350P = V350P*PK
C  ***************************************************
C COMPUTE DENSITY ANOMALY WITH RESPECT TO 1000.0 KG/M**3
C  1) DR350. DENSITY ANOMALY AT 35 (PSS-78), 0 DEG. C AND 0 DECIBARS
C  2) DR35P: DENSITY ANOMALY 35 (PSS-78), 0 DEG. C ,  PRES. VARIATION
C  3) DVAN : DENSITY ANOMALY VARIATIONS INVOLVING SPECFIC VOL. ANOMALY
C ***************************************************************
C CHECK VALUE: SIGMA = 59.82037  KG/M**3 FOR S = 40 (PSS-78),
C T = 40 DEG C, PO= 10000 DECIBARS.
C ********************************************** ************
      DR35P=GAM/V350P
      DVAN=SVA/(V350P*(V350P+SVA))
      SIGMA=DR350+DR35P-DVAN
      RETURN
      END
```

SPECIFIC VOLUME ANOMALY δ [1.0E-8 M**3/KG]

PRESSURE DECIBARS	TEMPERATURE °C IPTS-68 0	10	20	30	40	SALINITY: 0
0	2749.54	2763.60	2913.49	3170.58	3517.86	
1000	2692.64	2736.49	2904.38	3171.35	3522.43	
2000	2638.40	2710.21	2895.15	3171.60	3526.47	
3000	2586.70	2684.72	2885.75	3171.24	3529.86	
4000	2537.45	2659.94	2876.12	3170.22	3532.52	
5000	2490.57	2635.84	2866.23	3168.49	3534.39	
6000	2445.96	2612.37	2856.04	3166.00	3535.38	
7000	2403.53	2589.48	2845.51	3162.73	3535.46	
8000	2363.22	2567.14	2834.63	3158.63	3534.57	
9000	2324.94	2545.31	2823.37	3153.68	3532.67	
10000	2288.61	2523.95	2811.70	3147.85	3529.74	
						SALINITY: 30
0	383.22	480.62	681.41	967.03	1326.09	
1000	375.74	495.18	709.37	1002.13	1363.40	
2000	368.58	508.77	735.63	1035.19	1398.64	
3000	361.74	521.39	760.21	1066.25	1431.85	
4000	355.20	533.07	783.16	1095.35	1463.06	
5000	348.95	543.83	804.51	1122.54	1492.31	
6000	342.99	553.69	824.28	1147.84	1519.66	
7000	337.31	562.66	842.51	1171.31	1545.13	
8000	331.89	570.77	859.23	1192.97	1568.77	
9000	326.72	578.03	874.49	1212.86	1590.62	
10000	321.80	584.47	888.30	1231.03	1610.72	
						SALINITY: 35
0	0.00	109.29	317.33	607.14	968.21	
1000	0.00	130.28	351.06	647.58	1010.59	
2000	0.00	150.03	382.85	685.76	1050.67	
3000	0.00	168.57	412.76	721.75	1088.52	
4000	0.00	185.92	440.82	755.59	1124.19	
5000	0.00	202.12	467.08	787.33	1157.74	
6000	0.00	217.20	491.58	817.03	1189.23	
7000	0.00	231.18	514.37	844.73	1218.70	
8000	0.00	244.08	535.48	870.48	1246.22	
9000	0.00	255.95	554.95	894.34	1271.83	
10000	0.00	266.80	572.83	916.34	1295.58	
						SALINITY: 40
0	-380.79	-260.05	-45.03	248.83	611.92	
1000	-373.50	-232.77	-5.65	294.51	659.26	
2000	-366.52	-206.99	31.58	337.73	704.09	
3000	-359.84	-182.66	66.70	378.55	746.50	
4000	-353.46	-159.74	99.79	417.05	786.55	
5000	-347.35	-138.20	130.88	453.28	824.32	
6000	-341.52	-118.00	160.04	487.30	859.88	
7000	-335.96	-99.11	187.30	519.17	893.29	
8000	-330.64	-81.48	212.73	548.96	924.63	
9000	-325.58	-65.09	236.37	576.71	953.94	
10000	-320.74	-49.90	258.26	602.49	981.30	

DENSITY ANOMALY - [KG/M**3]

PRESSURE DECIBARS	TEMPERATURE °C IPTS-68 0	10	20	30	40	SALINITY: 0
0	-0.1574	-0.2979	-1.7937	-4.3489	-7.7796	
1000	4.8727	4.4302	2.7392	0.0620	-3.4369	
2000	9.7895	9.0578	7.1782	4.3817	0.8145	
3000	14.5955	13.5875	11.5264	8.6137	4.9785	
4000	19.2931	18.0221	15.7866	12.7611	9.0585	
5000	23.8849	22.3641	19.9618	16.8269	13.0578	
6000	28.3731	26.6162	24.0545	20.8142	16.9795	
7000	32.7602	30.7807	28.0675	24.7257	20.8267	
8000	37.0485	34.8600	32.0032	28.5641	24.6021	
9000	41.2400	38.8563	35.8640	32.3319	28.3086	
10000	45.3371	42.7718	39.6522	36.0315	31.9487	
0	24.0716	23.0511	20.9539	17.9854	14.2781	SALINITY: 30
1000	28.8258	27.5630	25.3064	22.2381	18.4767	
2000	33.4772	31.9820	29.5717	26.4061	22.5912	
3000	38.0283	36.3108	33.7524	30.4923	26.6245	
4000	42.4816	40.5521	37.8513	34.4994	30.5792	
5000	46.8396	44.7083	41.8710	38.4302	34.4580	
6000	51.1046	48.7820	45.8141	42.2871	38.2634	
7000	55.2790	52.7754	49.6829	46.0725	41.9978	
8000	59.3650	56.6909	53.4797	49.7888	45.6635	
9000	63.3647	60.5305	57.2067	53.4382	49.2627	
10000	67.2802	64.2965	60.8660	57.0228	52.7975	
0	28.1063	26.9524	24.7630	21.7286	17.9731	SALINITY: 35
1000	32.8184	31.4305	29.0871	25.9565	22.1496	
2000	37.4290	35.8168	33.3248	30.1005	26.2429	
3000	41.9407	40.1139	37.4788	34.1636	30.2558	
4000	46.3561	44.3245	41.5519	38.1484	34.1909	
5000	50.6777	48.4511	45.5467	42.0575	38.0507	
6000	54.9078	52.4963	49.4656	45.8933	41.8377	
7000	59.0487	56.4622	53.3110	49.6584	45.5541	
8000	63.1027	60.3513	57.0851	53.3548	49.2023	
9000	67.0719	64.1655	60.7902	56.9849	52.7842	
10000	70.9584	67.9071	64.4283	60.5506	56.3020	
0	32.1471	30.8624	28.5825	25.4829	21.6788	SALINITY: 40
1000	36.8179	35.3074	32.8787	29.6863	25.8336	
2000	41.3887	39.6615	37.0892	33.8068	29.9061	
3000	45.8620	43.9275	41.2170	37.8471	33.8990	
4000	50.2404	48.1080	45.2647	41.8099	37.8148	
5000	54.5263	52.2055	49.2348	45.6976	41.6560	
6000	58.7221	56.2226	53.1299	49.5128	45.4248	
7000	62.8302	60.1615	56.9521	53.2577	49.1236	
8000	66.8528	64.0244	60.7039	56.9345	52.7544	
9000	70.7920	67.8135	64.3873	60.5454	56.3194	
10000	74.6498	71.5310	68.0044	64.0924	59.8204	

SPECIFIC VOLUME V [1.0E-3 M**3/KG]

PRESSURE DECIBARS	TEMPERATURE °C IPTS-68 0	10	20	30	40	SALINITY: 0
0	1.0001575	1.0002980	1.0017970	1.0043678	1.0078406	
1000	0.9951509	0.9955893	0.9972682	0.9999380	1.0034488	
2000	0.9903054	0.9910235	0.9928730	0.9956374	0.9991861	
3000	0.9856145	0.9865947	0.9886050	0.9914599	0.9950461	
4000	0.9810721	0.9822970	0.9844587	0.9873998	0.9910228	
5000	0.9766723	0.9781251	0.9804289	0.9834515	0.9871105	
6000	0.9724097	0.9740738	0.9765105	0.9796102	0.9833040	
7000	0.9682789	0.9701384	0.9726987	0.9758709	0.9795982	
8000	0.9642751	0.9663143	0.9689891	0.9722292	0.9759886	
9000	0.9603934	0.9625970	0.9653777	0.9686807	0.9724707	
10000	0.9566292	0.9589825	0.9618601	0.9652216	0.9690405	
0	0.9764943	0.9774683	0.9794762	0.9823323	0.9859229	SALINITY: 30
1000	0.9719818	0.9731762	0.9753182	0.9782457	0.9818584	
2000	0.9676073	0.9690091	0.9712777	0.9742733	0.9779078	
3000	0.9633648	0.9649615	0.9673497	0.9704099	0.9740660	
4000	0.9592496	0.9610283	0.9635292	0.9666511	0.9703281	
5000	0.9552562	0.9572051	0.9598116	0.9629921	0.9666898	
6000	0.9513801	0.9534870	0.9561929	0.9594285	0.9631467	
7000	0.9476166	0.9498702	0.9526687	0.9559567	0.9596949	
8000	0.9439617	0.9463505	0.9492353	0.9525725	0.9563305	
9000	0.9404112	0.9429243	0.9458889	0.9492726	0.9530502	
10000	0.9369610	0.9395878	0.9426261	0.9460534	0.9498503	
0	0.9726620	0.9737550	0.9758353	0.9787335	0.9823442	SALINITY: 35
1000	0.9682245	0.9695272	0.9717350	0.9747003	0.9783304	
2000	0.9639214	0.9654217	0.9677499	0.9707791	0.9744282	
3000	0.9597476	0.9614332	0.9638751	0.9669650	0.9706328	
4000	0.9556976	0.9575568	0.9601058	0.9632535	0.9669394	
5000	0.9517666	0.9537879	0.9564374	0.9596400	0.9633441	
6000	0.9479501	0.9501221	0.9528660	0.9561205	0.9598424	
7000	0.9442436	0.9465553	0.9493873	0.9526908	0.9564306	
8000	0.9406430	0.9430838	0.9459977	0.9493477	0.9531051	
9000	0.9371440	0.9397034	0.9426935	0.9460874	0.9498623	
10000	0.9337431	0.9364110	0.9394714	0.9429065	0.9466990	
0	0.9688541	0.9700615	0.9722117	0.9751503	0.9787812	SALINITY: 40
1000	0.9644895	0.9658967	0.9681680	0.9711696	0.9748170	
2000	0.9602563	0.9618515	0.9642372	0.9672987	0.9709623	
3000	0.9561492	0.9579209	0.9604145	0.9635331	0.9672126	
4000	0.9521630	0.9541001	0.9566954	0.9598681	0.9635630	
5000	0.9482931	0.9503846	0.9530755	0.9562994	0.9600098	
6000	0.9445348	0.9467702	0.9495505	0.9528230	0.9565489	
7000	0.9408841	0.9432525	0.9461166	0.9494353	0.9531765	
8000	0.9373364	0.9398280	0.9427702	0.9461325	0.9498892	
9000	0.9338882	0.9364930	0.9395076	0.9429111	0.9466834	
10000	0.9305356	0.9332441	0.9363257	0.9397680	0.9435561	

4. Pressure to Depth Conversion

Saunders and Fofonoff (1976) developed an accurate formula for pressure to depth conversion using the hydrostatic equation and the Knudsen-Ekman equation of state. The formula included variation of gravity with latitude and depth. Depths estimated using the Saunders-Fofonoff formula deviate by only 0.08 m at 5000 decibars and by 0.44 m at 10000 decibars from estimates based on EOS80. These departures are considerably smaller than errors in pressure measurements.

However, for simplicity and consistency, a new formula (in SI units) based on EOS80 has been developed that is accurate to 0.1 m over the pressure range 0 - 10000 decibars. It should be noted that Saunders (1981) has developed a simpler but less precise quadratic depth formula based on EOS80.

Following Saunders and Fofonoff, the hydrostatic equation is integrated in the form

$$\int_p^z g\, dz = [g_o(\phi) + \tfrac{1}{2}\gamma z]\, z = \int_o^p V\, dp = \int_o^p V(35,0,p)\, dp + \Delta D \qquad (20)$$

where $g_o(\phi)$ is gravity at the ocean surface, a function of latitude and γ is is the mean vertical gradient of gravity, V is specific volume and ΔD geopotential anomaly determined by

$$\Delta D = \int_o^p \delta\, dp \qquad (21)$$

where

$$\delta = V(S,t,p) - V(35,0,p)$$

is specific volume anomaly.

EOS80 has the form

$$V(35,0,p) = V(35,0,0)\left(1 - \frac{p}{K + Ap + Bp^2}\right) \qquad (22)$$

which can be integrated exactly to yield

$$\int_o^p V(35,0,p)\, dp = V(35,0,0)\left[p - \frac{1}{2B}\left[\log_e (1 + Ap/K + Bp^2/K) + A/R \log_e \frac{(2BP + A - R)/(A-R)}{(2BP + A + R)/(A+R)}\right]\right] \qquad (23)$$

where $R^2 = A^2 - 4BK$ and

$$K = + 21582.27$$

$$A = + 3.35941$$

$$B = + 5.032 \quad E\text{-}5$$

$$V(35,0,0) = 9.72662\ E\text{-}4$$

for S = 35, t = 0°C and pressure in decibars.

Because the exact formula is not convenient for routine applications, a least squares polynomial of fourth order in pressure was fitted to a table of values computed from the exact formula over the range 0 - 12000 decibars with a precision equivalent to 0.0002 m or better, in the form

$$\int_0^p V(35,0,p)\, dp = C_1 p + C_2 p^2 + C_3 p^3 + C_4 p^4 \qquad (24)$$

where

$$C_1 = + 9.72659$$

$$C_2 = - 2.2512\ E\text{-}5$$

$$C_3 = +2.279\ E\text{-}10$$

$$C_4 = - 1.82 \quad E\text{-}15$$

Check value = 95224.4 for p = 10000 decibars.

The least squares formula eliminates the need for computing logarithms with negligible loss of precision. The complete formula has the form

$$z = \frac{C_1 p + C_2 p^2 + C_3 p^3 + C_4 p^4}{g(\phi) + \frac{1}{2}\gamma' p} + \frac{\Delta D}{9.8} \qquad (25)$$

$$\gamma' = + 2.184\ E\text{-}6 \qquad m/s^2/\text{decibar}$$

Gravity is the same as given in Saunder's and Fofonoff (1976) llowing Anon (1970) but with trigonometric substitutions for Sin(2ø).

$$g(\phi) = 9.780318\ (1.0 + 5.2788\ E\text{-}3\ \sin^2\phi + 2.36\ E\text{-}5\ \sin^4\phi)$$

Check value: $z = 9712.653$ m for $p = 10000$ decibars, Latitude $= 30^\circ$ where depth z is given in meters, pressure p in decibars, g in ms^{-2} and ΔD in J/kg. The correction for the actual density distribution is contained in the geopotential anomaly and is about 2 m or less. The replacement of depth by a linear pressure term in the depth correction to gravity yields an error of less than 0.1 m at 10000 decibars.

```
      REAL FUNCTION DEPTH(P,LAT)
C *********************************
C DEPTH IN METERS FROM PRESSURE IN DECIBARS USING
C SAUNDERS AND FOFONOFF'S METHOD.
C DEEP-SEA RES., 1976,23,109-111.
C FORMULA REFITTED FOR 1980 EQUATION OF STATE
C UNITS:
C       PRESSURE         P        DECIBARS
C       LATITUDE         LAT      DEGREES
C       DEPTH            DEPTH    METERS
C CHECKVALUE: DEPTH = 9712.653 M FOR P=10000 DECIBARS, LATITUDE=30 DEG
C     ABOVE FOR STANDARD OCEAN: T=0 DEG. CELSIUS ; S=35 (PSS-78)
C*********************************
      REAL LAT
C
      X = SIN(LAT/57.29578)
      X = X*X
C GR= GRAVITY VARIATION WITH LATITUDE: ANON (1970) BULLETIN GEODESIQUE
      GR = 9.780318*(1.0+(5.2788E-3+2.36E-5*X)*X) + 1.092E-6*P
      DEPTH = (((-1.82E-15*P+2.279E-10)*P-2.2512E-5)*P+9.72659)*P
      DEPTH=DEPTH/GR
      RETURN
      END
```

TABLE OF DEPTH (METERS)

PRESSURE DECIBARS	LATITUDE (DEGREES) 0	30	45	60	90
500	496.65	496.00	495.34	494.69	494.03
1000	992.12	990.81	989.50	988.19	986.88
2000	1979.55	1976.94	1974.33	1971.72	1969.11
3000	2962.43	2958.52	2954.61	2950.71	2946.81
4000	3940.88	3935.68	3930.49	3925.30	3920.10
5000	4915.04	4908.56	4902.08	4895.60	4889.13
6000	5885.03	5877.27	5869.51	5861.76	5854.01
7000	6850.95	6841.92	6832.89	6823.86	6814.84
8000	7812.93	7802.63	7792.33	7782.04	7771.76
9000	8771.07	8759.51	8747.95	8736.40	8724.85
10000	9725.47	9712.65	9699.84	9687.03	9674.23

5. Freezing Point Temperature of Seawater

A formula for computing freezing point temperatures of seawater was proposed by Millero (Millero and Leung, 1976) based on measurements by Doherty and Kester (1974) and adopted by the Joint Panel on Oceanographic Tables and Standards (Unesco Report No. 28, 1978, Annex 6).

The freezing point temperature t_f is given as a function of salinity S and pressure p by

$$t_f = a_0 S + a_1 S^{3/2} + a_2 S^2 + bp$$

where

$a_0 = -0.0575$

$a_1 = +1.710523 \text{ E-3}$

$a_2 = -2.154996 \text{ E-4}$

$b = -7.53 \text{ E-4}$

Check Value: $t_f = -2.588567^\circ C$ for $S = 40$, $p = 500$ decibars. The formula is valid in the range 4-40 practical salinity at atmospheric pressure. Measurements at elevated pressures showed no significant dependence of the pressure coefficient on salinity in the range 27 to 35 (Fujino, Lewis and Perkin, 1974). The estimated error to pressure of 500 decibars is $0.003^\circ C$ (Unesco Report No. 28, 1978, Annex 6).

```
      REAL FUNCTION TF(S,P)
C*************************************************************
C   FUNCTION TO COMPUTE THE FREEZING POINT OF SEAWATER
C
C   REFERENCE: UNESCO TECH. PAPERS IN THE MARINE SCIENCE NO. 28. 1978
C   EIGHTH REPORT JPOTS
C   ANNEX 6 FREEZING POINT OF SEAWATER F.J. MILLERO PP.29-35.
C
C  UNITS:
C         PRESSURE      P          DECIBARS
C         SALINITY      S          PSS-78
C         TEMPERATURE   TF         DEGREES CELSIUS
C         FREEZING PT.
C  CHECKVALUE: TF= -2.588567 DEG. C, FOR S=40.0, P=500. DECIBARS
C*************************************************************
      TF=(-.0575+1.710523E-3*SQRT(ABS(S))-2.154996E-4*S)*S-7.53E-4*P
      RETURN
      END
```

FREEZING POINT TEMPERATURE C

SALINITY PSS-78

PRESSURE DECIBARS	5	10	15	20	25	30	35	40
0	-0.274	-0.542	-0.812	-1.083	-1.358	-1.638	-1.922	-2.212
100	-0.349	-0.618	-0.887	-1.159	-1.434	-1.713	-1.998	-2.287
200	-0.424	-0.693	-0.962	-1.234	-1.509	-1.788	-2.073	-2.363
300	-0.500	-0.768	-1.038	-1.309	-1.584	-1.864	-2.148	-2.438
400	-0.575	-0.844	-1.113	-1.384	-1.660	-1.939	-2.224	-2.513
500	-0.650	-0.919	-1.188	-1.460	-1.735	-2.014	-2.299	-2.589

6. Specific Heat of Seawater

Specific heat of seawater C_p (J/(kg°C)), defined to be the heat in Joules required to raise the temperature of one kilogram of seawater one degree Celsius at constant pressure, is a function of salinity S, temperature t and pressure p. For seawater of oceanic salinities, the specific heat increases with temperature and decreases with salinity and pressure.

Cox and Smith (1959) measured C_p at atmospheric pressure for samples of Red Sea water diluted with distilled water over the salinity range 10-40°/•• (Knudsen scale) and temperature range 0-30°C (IPTS-48). More recently, Millero, Perron and Desnoyers (1973) measured C_p for standard seawater, diluted with pure water or concentrated by evaporation, over the chlorinity range 0-22°/•• and temperature range 5-35°C (IPTS-68). The agreement between these two sets of measurements and those of Bromley et al. (1967) was within ± 2 J/(kg°C) except at low temperatures where Cox's values are lower by as much as 6 J/(kg°C). Salinities converted from chlorinity are consistent with the PSS-78 salinities.

For computation of C_p, the empirical formula given by Millero et al. (1973) has been selected. The choice is guided by the better agreement between Millero and Bromley's results at low temperature and by their use of Standard Seawater for the measurements.

Specific heat values differ slightly on the IPTS-48 and IPTS-68 temperature scales because 1°C intervals on the two scales are different, i.e.,

$$\frac{dt_{48}}{dt_{68}} = 1.00044 \text{ at } 0°C$$

$$C_p(S,t_{68}) = C_p(S,t_{48}) \cdot \frac{dt_{48}}{dt_{68}} = C_p(S,t_{48}) \cdot 1.00044$$

so that for example

$$C_p(0,t_{48}) = 4217.4 \text{ J/(kg°C)}$$

$$C_p(0,t_{68}) = 4219.3 \text{ J/(kg°C)}$$

$$\Delta C_p = 1.9 \text{ J/(kg°C)} \quad \text{at } t_{48} = t_{68} = 0°C$$

The differences introduced by correcting for the temperature scale change are comparable to the accuracy of determination of C_p, and no correction is made for the possible change of temperature scale in the laboratory determinations. The effect on potential temperature calculations, for example, would not exceed 0.0004 °C and may be neglected.

The formula for C_p, converted to SI units and salinity, from Millero et al. (Unesco Report No. 38 1981)

$$C_p(S,t,0) = C_p(0,t,0) + AS + BS^{3/2} \tag{26}$$

where

$$C_p(0,t,0) = C_0 + C_1 t + C_2 t^2 + C_3 t^3 + C_4 t^4$$

$$A = a_0 + a_1 t + a_2 t^2$$

$$B = b_0 + b_1 t + b_2 t^2$$

C_0	= + 4217.4	a_0	= - 7.643575	b_0	= + 0.1770383
C_1	= - 3.720283	a_1	= + 0.1072763	b_1	= - 4.07718 E-3
C_2	= + 0.1412855	a_2	= - 1.38385 E-3	b_2	= + 5.148 E-5
C_3	= - 2.654387 E-3				
C_4	= + 2.093236 E-5				

Range of Validity: S = 0 to 40 ; t = 0 to 35°C

Check: C_p = 3980.051 J/(kg°C) for S = 40, t = 40°C, Std. Dev.= 0.5 J/(kg°C)

Specific heat of seawater C_p: Pressure Dependence

Direct measurements of specific heat of seawater are not available. The pressure dependence is computed from the thermodynamic equation

$$\frac{\partial C_p}{\partial p} = -T \frac{\partial^2 V}{\partial t^2}$$

where V is specific volume (m^3/kg), T absolute temperature (K) and p pressure (P_a). For pressure in bars, the equation can be integrated to yield

$$C_p(S,t,p) = C_p(S,t,0) - 10^5 \int_0^p (t + 273.15) \frac{\partial^2 V}{\partial t^2} dp \,. \qquad (27)$$

Because the pressure dependence is not economically evaluated from the exact integrals, a polynomial expression has been fitted to a table of values generated from the exact formulas. The least squares formula has been fitted in two steps. Polynomials in temperature and pressure were fitted to values of $\Delta_1 C_p = C_p(0,t,p) - C_p(0,t,0)$ to obtain the pressure dependence at S = 0. The salinity and pressure dependence was obtained by fitting values of $\Delta_2 C_p = C_p(S,t,p) - C_p(S,t,0) - [C_p(0,t,p) - C(0,t,0)]$. The two specific heat pressure polynomials $\Delta_1 C_p$ and $\Delta_2 C_p$ are valid over the range of salinity, temperature, and pressure of EOS80. These least squares polynomials combined with the Millero et al. (1981) polynomials for $C_p(S,t,0)$ provide specific heat estimates over the full range of salinity, temperature and pressure.

Specific heat of seawater C_p

$$C_p(S,t,p) = C_p(S,t,0) + \Delta_1 C_p(0,t,p) + \Delta_2 C_p(S,t,p)$$

Polynomials for S = 0

$$\begin{aligned} \Delta_1 C_p(0,t,p) = {} & (a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4)\, p \\ & + (b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4)\, p^2 \\ & + (c_0 + c_1 t + c_2 t^2 + c_3 t^3)\, p^3 \end{aligned} \qquad (28)$$

$a_0 = -4.9592\ E\text{-}1$ $\quad b_0 = +2.4931\ E\text{-}4$

$a_1 = +1.45747\ E\text{-}2$ $\quad b_1 = -1.08645\ E\text{-}5$

$a_2 = -3.13885\ E\text{-}4$ $\quad b_2 = +2.87533\ E\text{-}7$

$a_3 = +2.0357\ E\text{-}6$ $\quad b_3 = -4.0027\ E\text{-}9$

$a_4 = +1.7168\ E\text{-}8$ $\quad b_4 = +2.2956\ E\text{-}11$

$c_0 = -5.422\ E\text{-}8$

$c_1 = +2.6380\ E\text{-}9$

$c_2 = -6.5637\ E\text{-}11$

$c_3 = +6.136\ E\text{-}13$

CHECK VALUES

$C_p(S,T,0) = 3980.051\ J/(kg^\circ C)$

$\Delta_1 C_p(0,T,P) = -177.985\ J/(kg^\circ C)$

$p = 10000$ decibars, $t = 40^\circ C$ $S = 40$

Standard Deviation: $\Delta_1 C_p = 0.074\ J/(kg^\circ C)$

Polynomials for $S > 0$

$$\begin{aligned}\Delta_2 C_p(S,T,P) = {} & [(d_0 + d_1 t + d_2 t^2 + d_3 t^3 + d_4 t^4)\, S \\ & + (e_0 + e_1 t + e_2 t^2)\, S^{3/2}]\, p \\ & + [(f_0 + f_1 t + f_2 t^2 + f_3 t^3)\, S \\ & + (g_0)\, S^{3/2}]\, p^2 \\ & + [(h_0 + h_1 t + h_2 t^2)\, S \\ & + J_1 t\, S^{3/2}]\, p^3\end{aligned} \qquad (29)$$

d_0 = + 4.9247 E-3

d_1 = - 1.28315E-4

d_2 = + 9.802 E-7

d_3 = + 2.5941 E-8

d_4 = - 2.9179 E-10

e_0 = - 1.2331 E-4

e_1 = - 1.517 E-6

e_2 = + 3.122 E-8

f_0 = - 2.9558 E-6

f_1 = + 1.17054E-7

f_2 = - 2.3905 E-9

f_3 = + 1.8448 E-11

g_0 = + 9.971 E-8

h_0 = + 5.540 E-10

h_1 = - 1.7682 E-11

h_2 = + 3.513 E-13

J_1 = - 1.4300 E-12

CHECK VALUE

$\Delta_2 C_p(S,t,p)$ = 47.433 J/(kg°C)

$C_p(S,t,p)$ = 3849.500 J/(kg°C)

p = 10000 decibars, t = 40°C S = 40

Standard Deviation: $\Delta_2 C_p$ = .062 J/(kg°C)

```
      REAL FUNCTION CPSW(S,T,PO)
C ****************************
C UNITS:
C       PRESSURE        PO       DECIBARS
C       TEMPERATURE     T        DEG CELSIUS (IPTS-68)
C       SALINITY        S        (PSS-78)
C       SPECIFIC HEAT   CPSW     J/(KG DEG C)
C*************************************************************
C REF: MILLERO ET AL,1973,JGR,78,4499-4507
C       MILLERO ET AL, UNESCO REPORT NO. 38 1981 PP. 99-188.
C PRESSURE VARIATION FROM LEAST SQUARES POLYNOMIAL
C DEVELOPED BY FOFONOFF 1980.
C CHECK VALUE: CPSW = 3849.500 J/(KG DEG. C) FOR S = 40 (PSS-78),
C T = 40 DEG C, PO= 10000 DECIBARS
C***************************
C   SCALE PRESSURE TO BARS
      P=PO/10.
C SQRT SALINITY FOR FRACTIONAL TERMS
      SR = SQRT(ABS(S))
C SPECIFIC HEAT CPO FOR P=0 (MILLERO ET AL, UNESCO REPORT NO. 38 1981)
      A = (-1.38385E-3*T+0.1072763)*T-7.643575
      B = (5.148E-5*T-4.07718E-3)*T+0.1770383
      C = (((2.093236E-5*T-2.654387E-3)*T+0.1412855)*T
     X    -3.720283)*T+4217.4
      CPO = (B*SR + A)*S + C
C CP1 PRESSURE AND TEMPERATURE TERMS FOR S = 0
      A = (((1.7168E-8*T+2.0357E-6)*T-3.13885E-4)*T+1.45747E-2)*T
     X   -0.49592
      B = (((2.2956E-11*T-4.0027E-9)*T+2.87533E-7)*T-1.08645E-5)*T
     X   +2.4931E-4
      C = ((6.136E-13*T-6.5637E-11)*T+2.6380E-9)*T-5.422E-8
      CP1 = ((C*P+B)*P+A)*P
C CP2 PRESSURE AND TEMPERATURE TERMS FOR S > 0
      A = (((-2.9179E-10*T+2.5941E-8)*T+9.802E-7)*T-1.28315E-4)*T
     X   +4.9247E-3
      B = (3.122E-8*T-1.517E-6)*T-1.2331E-4
      A = (A+B*SR)*S
      B = ((1.8448E-11*T-2.3905E-9)*T+1.17054E-7)*T-2.9558E-6
      B = (B+9.971E-8*SR)*S
      C = (3.513E-13*T-1.7682E-11)*T+5.540E-10
      C = (C-1.4300E-12*T*SR)*S
      CP2 = ((C*P+B)*P+A)*P
C SPECIFIC HEAT RETURN
      CPSW = CPO + CP1 + CP2
      RETURN
      END
```

SPECIFIC HEAT SEAWATER C_p [J/(KG°C)]

PRESSURE DECIBARS	TEMPERATURE °C IPTS-68 0	10	20	30	40	
0	4048.4	4041.8	4044.8	4049.1	4051.2	SALINITY: 25
1000	4011.5	4012.9	4020.2	4026.9	4031.8	
2000	3978.0	3986.3	3997.4	4006.2	4013.6	
3000	3947.8	3962.0	3976.2	3986.9	3996.7	
4000	3920.6	3939.8	3956.7	3968.9	3980.9	
5000	3896.3	3919.6	3938.6	3952.0	3966.1	
6000	3874.4	3901.1	3921.9	3936.3	3952.4	
7000	3854.9	3884.3	3906.5	3921.7	3939.5	
8000	3837.4	3869.0	3892.2	3907.9	3927.6	
9000	3821.8	3855.1	3879.0	3895.1	3916.4	
10000	3807.7	3842.3	3866.7	3883.0	3905.9	
0	4017.2	4013.8	4019.1	4024.7	4027.2	SALINITY: 30
1000	3982.1	3986.2	3995.4	4003.2	4008.4	
2000	3950.3	3960.8	3973.3	3983.1	3990.8	
3000	3921.6	3937.6	3953.0	3964.3	3974.4	
4000	3895.7	3916.3	3934.1	3946.9	3959.1	
5000	3872.5	3897.0	3916.7	3930.6	3944.9	
6000	3851.7	3879.3	3900.6	3915.3	3931.7	
7000	3833.1	3863.3	3885.7	3901.1	3919.3	
8000	3816.5	3848.7	3872.0	3887.8	3907.9	
9000	3801.6	3835.4	3859.2	3875.4	3897.1	
10000	3788.2	3823.2	3847.4	3863.6	3887.1	
0	3986.5	3986.3	3993.9	4000.7	4003.5	SALINITY: 35
1000	3953.3	3959.9	3970.9	3979.7	3985.2	
2000	3923.1	3935.7	3949.6	3960.2	3968.2	
3000	3895.9	3913.5	3930.0	3942.0	3952.3	
4000	3871.3	3893.2	3911.8	3925.1	3937.6	
5000	3849.3	3874.7	3895.0	3909.2	3923.9	
6000	3829.5	3857.9	3879.5	3894.5	3911.1	
7000	3811.8	3842.6	3865.2	3880.7	3899.3	
8000	3796.0	3828.7	3851.9	3867.8	3888.2	
9000	3781.8	3816.0	3839.7	3855.7	3877.9	
10000	3769.1	3804.4	3828.3	3844.3	3868.3	
0	3956.4	3959.3	3968.9	3977.0	3980.1	SALINITY: 40
1000	3925.0	3934.1	3946.8	3956.6	3962.3	
2000	3896.4	3910.9	3926.2	3937.6	3945.8	
3000	3870.6	3889.8	3907.2	3919.9	3930.5	
4000	3847.4	3870.4	3889.7	3903.4	3916.2	
5000	3826.4	3852.8	3873.5	3888.1	3903.0	
6000	3807.7	3836.7	3858.6	3873.8	3890.7	
7000	3790.9	3822.2	3844.8	3860.4	3879.3	
8000	3776.0	3808.9	3832.0	3847.9	3868.7	
9000	3762.6	3796.9	3820.2	3836.1	3858.8	
10000	3750.6	3785.9	3809.3	3825.1	3849.5	

7. Adiabatic Lapse Rate

The adiabatic lapse rate $\Gamma(S,t,p)$ (°C/decibar) is defined as the change of temperature per unit pressure for an adiabatic change of pressure of an element of seawater. It is assumed that no heat or salt is exchanged with the surroundings so that the pressure change is both adiabatic and isentropic. From thermodynamic considerations, the adiabatic lapse rate Γ, a function of pressure, temperature and salinity can be expressed as

$$\Gamma(S,t,p) = \frac{T\partial V/\partial t}{C_p} \qquad (30)$$

where $T = t + 273.15$ is absolute temperature (Kelvin), $\partial V/\partial t$ ($m^3/(kg\ °C)$) is thermal expansion and C_p ($J/(kg\ °C)$) specific heat of seawater at constant pressure.

The lapse rate Γ is positive except at low salinities, temperatures and pressures where $\partial V/\partial t$ is negative. Typical values in the oceanic range are $1\text{-}2 \times 10^{-4}$ °C/decibar.

Adiabatic lapse rate can be calculated from the equation of state and specific heat (Bryden, 1973) or from direct measurements (Caldwell and Eide, 1980).

Caldwell and Eide (1980) measured the temperature change resulting from an adiabatic pressure change directly and constructed formulas to estimate lapse rate and potential temperature. Agreement with estimates based on the thermodynamic equations using EOS80 and Millero *et al.* (1973) specific heats is within 5×10^{-3} °C at 35 over the pressure range 0-10000 decibars except at low temperatures (< 2°C) and high pressures (> 4000 decibars) where differences exceed 10×10^{-3} °C. As these differences are within the stated error of their measurements and within the estimated accuracy of the thermodynamic values (Bryden, 1973) the relative accuracy of the two estimates is indeterminate.

Bryden's (1973) formulas are selected because of their closer agreement with values obtained from EOS80. These formulas yield values for potential temperature within 6×10^{-3} °C over the entire pressure and temperature range at a salinity of 35.

$$\Gamma(S,t,p) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + (b_0 + b_1 t)(S-35) + [c_0 + c_1 t + c_2 t^2 + c_3 t^3 + (d_0 + d_1 t)(S-35)]\, p + [e_0 + e_1 t + e_2 t^2]\, p^2 \qquad (31)$$

$a_0 = +\ 3.5803\ E\text{-}5$

$a_1 = +\ 8.5258\ E\text{-}6$

$a_2 = -\ 6.8360\ E\text{-}8$

$a_3 = +\ 6.6228\ E\text{-}10$

$b_0 = +\ 1.8932\ E\text{-}6$

$b_1 = -\ 4.2393\ E\text{-}8$

$c_0 = +\ 1.8741\ E\text{-}8$

$c_1 = -\ 6.7795\ E\text{-}10$

$c_2 = +\ 8.7330\ E\text{-}12$

$c_3 = -\ 5.4481\ E\text{-}14$

$d_0 = -\ 1.1351\ E\text{-}10$

$d_1 = +\ 2.7759\ E\text{-}12$

$e_0 = -\ 4.6206\ E\text{-}13$

$e_1 = +\ 1.8676\ E\text{-}14$

$e_2 = -\ 2.1687\ E\text{-}16$

CHECK VALUE:

$\Gamma = 3.255976\ E\text{-}4$ °C /decibar

For $S = 40$ $t = 40$ °C $p = 10000$ decibars

Standard Deviation: 3.38×10^{-7} °C/decibar

Range of Validity: The ranges with differences less than one percent from direct calculations using EOS80 are outlined in the adiabatic lapse rate table.

```
C  SEPT 25 1982
      REAL FUNCTION ATG(S,T,P)
C ****************************
C ADIABATIC TEMPERATURE GRADIENT DEG C PER DECIBAR
C REF: BRYDEN,H.,1973,DEEP-SEA RES.,20,401-408
C UNITS:
C       PRESSURE         P        DECIBARS
C       TEMPERATURE      T        DEG CELSIUS (IPTS-68)
C       SALINITY         S        (PSS-78)
C       ADIABATIC        ATG      DEG. C/DECIBAR
C CHECKVALUE: ATG=3.255976E-4 C/DBAR FOR S=40 (PSS-78),
C T=40 DEG C,P=10000 DECIBARS
C ****************************
      DS = S - 35.0
      ATG = (((-2.1687E-16*T+1.8676E-14)*T-4.6206E-13)*P
     X+((2.7759E-12*T-1.1351E-10)*DS+((-5.4481E-14*T
     X+8.733E-12)*T-6.7795E-10)*T+1.8741E-8))*P
     X+(-4.2393E-8*T+1.8932E-6)*DS
     X+((6.6228E-10*T-6.836E-8)*T+8.5258E-6)*T+3.5803E-5
      RETURN
      END
```

ADIABATIC LAPSE RATE Γ [°C/1000 Decibars]

PRESSURE DECIBARS	TEMPERATURE °C IPTS-68 0	10	20	30	40	SALINITY: 25
0	0.0169	0.1002	0.1738	0.2417	0.3079	
1000	0.0363	0.1135	0.1825	0.2467	0.3099	
2000	0.0548	0.1263	0.1908	0.2515	0.3119	
3000	0.0723	0.1384	0.1987	0.2561	0.3137	
4000	0.0890	0.1500	0.2063	0.2605	0.3154	
5000	0.1047	0.1610	0.2135	0.2648	0.3170	
6000	0.1195	0.1713	0.2204	0.2688	0.3184	
7000	0.1334	0.1811	0.2270	0.2726	0.3198	
8000	0.1463	0.1903	0.2332	0.2763	0.3210	
9000	0.1583	0.1989	0.2390	0.2797	0.3221	
10000	0.1694	0.2069	0.2445	0.2830	0.3230	
						SALINITY: 30
0	0.0263	0.1075	0.1790	0.2448	0.3089	
1000	0.0452	0.1205	0.1874	0.2497	0.3109	
2000	0.0631	0.1328	0.1954	0.2543	0.3129	
3000	0.0801	0.1445	0.2031	0.2588	0.3147	
4000	0.0962	0.1556	0.2104	0.2630	0.3164	
5000	0.1113	0.1662	0.2173	0.2671	0.3179	
6000	0.1256	0.1761	0.2239	0.2710	0.3194	
7000	0.1389	0.1855	0.2302	0.2747	0.3207	
8000	0.1512	0.1942	0.2361	0.2782	0.3219	
9000	0.1627	0.2024	0.2416	0.2815	0.3229	
10000	0.1732	0.2099	0.2468	0.2846	0.3239	
						SALINITY: 35
0	0.0358	0.1149	0.1843	0.2479	0.3098	
1000	0.0541	0.1274	0.1923	0.2526	0.3119	
2000	0.0714	0.1393	0.2001	0.2571	0.3138	
3000	0.0879	0.1506	0.2074	0.2614	0.3156	
4000	0.1034	0.1613	0.2144	0.2655	0.3173	
5000	0.1180	0.1714	0.2211	0.2695	0.3188	
6000	0.1316	0.1809	0.2274	0.2732	0.3203	
7000	0.1443	0.1898	0.2334	0.2767	0.3216	
8000	0.1562	0.1981	0.2390	0.2801	0.3227	
9000	0.1670	0.2059	0.2442	0.2832	0.3238	
10000	0.1770	0.2130	0.2491	0.2862	0.3247	
						SALINITY: 40
0	0.0453	0.1222	0.1895	0.2510	0.3108	
1000	0.0630	0.1343	0.1973	0.2556	0.3129	
2000	0.0798	0.1457	0.2047	0.2599	0.3148	
3000	0.0956	0.1566	0.2118	0.2641	0.3166	
4000	0.1106	0.1669	0.2185	0.2681	0.3182	
5000	0.1246	0.1766	0.2249	0.2718	0.3198	
6000	0.1377	0.1857	0.2309	0.2754	0.3212	
7000	0.1498	0.1941	0.2366	0.2788	0.3225	
8000	0.1611	0.2020	0.2419	0.2820	0.3236	
9000	0.1714	0.2093	0.2468	0.2849	0.3247	
10000	0.1808	0.2161	0.2515	0.2877	0.3256	

8. Potential Temperature

Potential temperature has been defined classically as the temperature an element of seawater would have if raised adiabatically with no change of salinity to atmospheric pressure. More generally, the potential temperature can be defined as the temperature resulting from an adiabatic displacement to a reference pressure P_r that may be greater or less than the initial pressure p. The potential temperature θ can be computed from the adiabatic lapse rate Γ,

$$\theta(S,t,p,p_r) = t + \int_{p}^{p_r} \Gamma(S,t',p)\, dp' \qquad (32)$$

by integration along an adiabat, i.e., $\partial t'/\partial p' = \Gamma'$. The potential temperature can be evaluated by using an empirical formula or by numerical integration of the defining equation. Only the numerical integration by the Runge-Kutta method is given as both it and a Newton-Raphson iterative technique of the Bryden potential temperature at atmospheric pressure require about the same number of calculations and the latter retains the fitting errors of the potential temperature formula.

The potential temperature $\theta(S,t,p,p_r)$ at reference pressure p_r can be computed with sufficient precision using a 4th order Runge-Kutta integration algorithm (Fofonoff, 1977). If S_0, t_0, p_0 are the *in situ* values, the potential temperature θ at p_r is estimated as follows:

$$\Delta\theta_1 = \Delta p \cdot \Gamma(S_0, t_0, p_0) \qquad \theta_1 = t_0 + \tfrac{1}{2}\Delta\theta_1$$

$$\Delta\theta_2 = \Delta p \cdot \Gamma(S_0, \theta_1, p_0 + \tfrac{1}{2}\Delta p) \qquad \theta_2 = \theta_1 + (1 - 1/\sqrt{2})(\Delta\theta_2 - q_1)$$

$$\Delta\theta_3 = \Delta p \cdot \Gamma(S_0, \theta_2, p_0 + \tfrac{1}{2}\Delta p) \qquad \theta_3 = \theta_2 + (1 + 1/\sqrt{2})(\Delta\theta_3 - q_2)$$

$$\Delta\theta_4 = \Delta p \cdot \Gamma(S_0, \theta_3, p_0 + \Delta p) \qquad \theta_4 = \theta_3 + \tfrac{1}{6}(\Delta\theta_4 - 2q_3)$$

where $q_1 = \Delta\theta_1$

$$q_2 = (2 - \sqrt{2})\ \Delta\theta_2 + (-2 + 3/\sqrt{2})\ q_1$$

$$q_3 = (2 + \sqrt{2})\ \Delta\theta_3 + (-2 - 3/\sqrt{2})\ q_2$$

$$\Delta p = p_r - p_0$$

and $\theta(S_0 t_0, p_0, p_r) = \theta_4$

The integration error is less than 0.1×10^{-3} °C for Δp = 10000 decibars
Check Value: θ = 36.89073 °C for S = 40, t = 40 °C, p = 10000 decibars,

p_r = 0 decibars

Range of Validity: The ranges with differences less than 0.005°C from direct calculations using EOS80 are outlined in the potential temperature table.

```
      REAL FUNCTION THETA(S,TO,PO,PR)
C *************************************
C TO COMPUTE LOCAL POTENTIAL TEMPERATURE AT PR
C USING BRYDEN 1973 POLYNOMIAL FOR ADIABATIC LAPSE RATE
C AND RUNGE-KUTTA 4-TH ORDER INTEGRATION ALGORITHM.
C REF: BRYDEN,H.,1973,DEEP-SEA RES.,20,401-408
C FOFONOFF,N.,1977,DEEP-SEA RES.,24,489-491
C UNITS:
C       PRESSURE         PO       DECIBARS
C       TEMPERATURE      TO       DEG CELSIUS (IPTS-68)
C       SALINITY         S        (PSS-78)
C       REFERENCE PRS    PR       DECIBARS
C       POTENTIAL TMP.   THETA    DEG CELSIUS
C CHECKVALUE: THETA= 36.89073 C,S=40 (PSS-78),TO=40 DEG C,
C PO=10000 DECIBARS,PR=0 DECIBARS
C *************************************
C      SET-UP INTERMEDIATE TEMPERATURE AND PRESSURE VARIABLES
      P=PO
      T=TO
      H = PR - P
      XK = H*ATG(S,T,P)
      T = T + 0.5*XK
      Q = XK
      P = P + 0.5*H
      XK = H*ATG(S,T,P)
      T = T + 0.29289322*(XK-Q)
      Q = 0.58578644*XK + 0.121320344*Q
      XK = H*ATG(S,T,P)
      T = T + 1.707106781*(XK-Q)
      Q = 3.414213562*XK - 4.121320344*Q
      P = P + 0.5*H
      XK = H*ATG(S,T,P)
      THETA = T + (XK-2.0*Q)/6.0
      RETURN
      END
```

POTENTIAL TEMPERATURE θ °C (Ref. Pres. = 0.0)

PRESSURE DECIBARS	TEMPERATURE °C IPTS-68 0	10	20	30	40
					SALINITY: 25
0	0.0000	10.0000	20.0000	30.0000	40.0000
1000	-0.0265	9.8935	19.8224	29.7566	39.6921
2000	-0.0715	9.7749	19.6377	29.5098	39.3842
3000	-0.1339	9.6448	19.4461	29.2599	39.0763
4000	-0.2125	9.5040	19.2480	29.0070	38.7684
5000	-0.3061	9.3531	19.0438	28.7512	38.4607
6000	-0.4139	9.1927	18.8338	28.4928	38.1531
7000	-0.5349	9.0234	18.6183	28.2319	37.8458
8000	-0.6679	8.8459	18.3978	27.9685	37.5387
9000	-0.8122	8.6607	18.1724	27.7030	37.2319
10000	-0.9667	8.4684	17.9426	27.4353	36.9254
					SALINITY: 30
0	0.0000	10.0000	20.0000	30.0000	40.0000
1000	-0.0357	9.8864	19.8174	29.7535	39.6911
2000	-0.0892	9.7611	19.6278	29.5038	39.3821
3000	-0.1595	9.6248	19.4317	29.2511	39.0731
4000	-0.2454	9.4782	19.2293	28.9955	38.7641
5000	-0.3458	9.3219	19.0211	28.7372	38.4552
6000	-0.4598	9.1565	18.8074	28.4763	38.1465
7000	-0.5864	8.9827	18.5885	28.2130	37.8379
8000	-0.7246	8.8010	18.3648	27.9475	37.5296
9000	-0.8734	8.6120	18.1365	27.6798	37.2216
10000	-1.0320	8.4164	17.9040	27.4102	36.9139
					SALINITY: 35
0	0.0000	10.0000	20.0000	30.0000	40.0000
1000	-0.0448	9.8793	19.8123	29.7505	39.6901
2000	-0.1069	9.7473	19.6179	29.4979	39.3800
3000	-0.1851	9.6048	19.4172	29.2423	39.0699
4000	-0.2783	9.4523	19.2106	28.9840	38.7598
5000	-0.3856	9.2906	18.9985	28.7231	38.4498
6000	-0.5058	9.1203	18.7810	28.4598	38.1399
7000	-0.6380	8.9419	18.5587	28.1942	37.8301
8000	-0.7813	8.7560	18.3317	27.9264	37.5205
9000	-0.9348	8.5633	18.1005	27.6567	37.2113
10000	-1.0974	8.3643	17.8654	27.3851	36.9023
					SALINITY: 40
0	0.0000	10.0000	20.0000	30.0000	40.0000
1000	-0.0540	9.8721	19.8072	29.7474	39.6891
2000	-0.1246	9.7335	19.6080	29.4919	39.3780
3000	-0.2107	9.5847	19.4028	29.2335	39.0668
4000	-0.3113	9.4265	19.1919	28.9725	38.7556
5000	-0.4253	9.2594	18.9758	28.7091	38.4444
6000	-0.5518	9.0840	18.7546	28.4433	38.1332
7000	-0.6897	8.9011	18.5288	28.1753	37.8223
8000	-0.8381	8.7110	18.2986	27.9053	37.5115
9000	-0.9962	8.5145	18.0645	27.6335	37.2009
10000	-1.1629	8.3121	17.8266	27.3599	36.8907

9. Sound Speed in Seawater

Sound speed has been measured for samples of standard seawater, diluted with pure water or concentrated by evaporation, by Chen and Millero (1977). The formula developed from these measurements is consistent with the new salinity scale and is in better agreement with values computed from EOS80 than the formulas given by Wilson (1960) (see the following Sound Speed Comparison Table) and Del Grosso and Mader (1972).

Speed of sound in seawater U (Chen and Millero, 1977)

$$U(S,t,p) = C_w(t,p) + A(t,p)S + B(t,p)\,S^{3/2} + D(t,p)S^2 \qquad (33)$$

$$\begin{aligned} C_w(t,p) = {} & C_{00} + C_{01}t + C_{02}t^2 + C_{03}t^3 + C_{04}t^4 + C_{05}t^5 \\ & + (C_{10} + C_{11}t + C_{12}t^2 + C_{13}t^3 + C_{14}t^4)\ p \\ & + (C_{20} + C_{21}t + C_{22}t^2 + C_{23}t^3 + C_{24}t^4)\ p^2 \\ & + (C_{30} + C_{31}t + C_{32}t^2)\ p^3 \end{aligned} \qquad (34)$$

C_{00} = + 1402.388

C_{01} = + 5.03711

C_{02} = - 5.80852 E-2

C_{03} = + 3.3420 E-4

C_{04} = - 1.47800 E-6

C_{05} = + 3.1464 E-9

C_{10} = + 0.153563

C_{11} = + 6.8982 E-4

C_{12} = - 8.1788 E-6

C_{13} = + 1.3621 E-7

C_{14} = - 6.1185 E-10

C_{20} = + 3.1260 E-5

C_{21} = - 1.7107 E-6

C_{22} = + 2.5974 E-8

C_{23} = - 2.5335 E-10

C_{24} = + 1.0405 E-12

C_{30} = - 9.7729 E-9

C_{31} = + 3.8504 E-10

C_{32} = - 2.3643 E-12

$$A(t,p) = A_{00} + A_{01}t + A_{02}t^2 + A_{03}t^3 + A_{04}t^4 + (A_{10} + A_{11}t + A_{12}t^2 + A_{13}t^3 + A_{14}t^4)\,p + (A_{20} + A_{21}t + A_{22}t^2 + A_{23}t^3)\,p^2 + (A_{30} + A_{31}t + A_{32}t^2)\,p^3 \quad (35)$$

A_{00} = + 1.389		A_{10} = + 9.4742	E-5
A_{01} = - 1.262	E-2	A_{11} = - 1.2580	E-5
A_{02} = + 7.164	E-5	A_{12} = - 6.4885	E-8
A_{03} = + 2.006	E-6	A_{13} = + 1.0507	E-8
A_{04} = - 3.21	E-8	A_{14} = - 2.0122	E-10
A_{20} = - 3.9064	E-7	A_{30} = + 1.100	E-10
A_{21} = + 9.1041	E-9	A_{31} = + 6.649	E-12
A_{22} = - 1.6002	E-10	A_{32} = - 3.389	E-13
A_{23} = + 7.988	E-12		

$$B(t,p) = B_{00} + B_{01}t + (B_{10} + B_{11}t)\,p \quad (36)$$

B_{00} = - 1.922	E-2	B_{10} = + 7.3637	E-5
B_{01} = - 4.42	E-5	B_{11} = + 1.7945	E-7

$$D(t,p) = D_{00} + D_{10}\,p \quad (37)$$

D_{00} = + 1.727	E-3	D_{10} = - 7.9836	E-6

Range of validity: S = 0 to 40; t = 0 to 40°C; p = 0 to 10000 decibars

Standard Deviation: 0.19 m/s

U(S,t,0) - U(0,t,0): .03 m/s Millero and Kubinski (1975)

U(S,t,p) - U(0,t,p) - (U(S,t,0) - U(0,t,0)): 0.19 Chen and Millero (1976)

U(0,t,p) - U(0,t,0): .08 m/s Chen and Millero (1976)

Check Value: 1731.995 m/s for S = 40, t = 40°C, p = 10000 decibars

Sound Speed Comparison Table[1]

(a) Chen/Millero (1977) Formula - U(EOS80)

S:	0	5	10	15	20	25	30	35	40	Salinity
Δ:	.15	.24	.32	.39	.42	.42	.38	.30	.18	m/s
σ:	.20	.20	.23	.27	.30	.31	.31	.32	.40	m/s

(b) Wilson (1960) Formula - U(EOS80)

S:	0	5	10	15	20	25	30	35	40	Salinity
Δ:	1.10	.85	.65	.51	.48	.49	.60	.82	1.13	m/s
σ:	1.15	1.04	.96	.89	.85	.80	.77	.75	.75	m/s

Δ = mean difference
σ = standard deviation

[1]The differences (formula indicated minus sound speed computed using EOS80) at each salinity in the Sound Speed Comparison Table above are computed from 242 values over the temperature range -2 to 40°C at 2°C intervals and pressure range 0 to 10000 decibars at 1000 decibar intervals.

```
      REAL FUNCTION SVEL(S,T,PO)
C *******************************
C SOUND SPEED SEAWATER CHEN AND MILLERO 1977,JASA,62,1129-1135
C UNITS:
C       PRESSURE         PO       DECIBARS
C       TEMPERATURE      T        DEG CELSIUS (IPTS-68)
C       SALINITY         S        (PSS-78)
C       SOUND SPEED      SVEL     METERS/SECOND
C CHECKVALUE: SVEL=1731.995 M/S, S=40 (PSS-78),T=40 DEG C,P=10000 DBAR
C *******************************
      EQUIVALENCE (A0,B0,C0),(A1,B1,C1),(A2,C2),(A3,C3)
C
C   SCALE PRESSURE TO BARS
      P=PO/10.
      SR = SQRT(ABS(S))
C S**2 TERM
      D = 1.727E-3 - 7.9836E-6*P
C S**3/2 TERM
      B1 = 7.3637E-5 +1.7945E-7*T
      B0 = -1.922E-2 -4.42E-5*T
      B = B0 + B1*P
C S**1 TERM
      A3 = (-3.389E-13*T+6.649E-12)*T+1.100E-10
      A2 = ((7.988E-12*T-1.6002E-10)*T+9.1041E-9)*T-3.9064E-7
      A1 = (((-2.0122E-10*T+1.0507E-8)*T-6.4885E-8)*T-1.2580E-5)*T
     X     +9.4742E-5
      A0 = (((-3.21E-8*T+2.006E-6)*T+7.164E-5)*T-1.262E-2)*T
     X     +1.389
      A = ((A3*P+A2)*P+A1)*P+A0
C S**0 TERM
      C3 = (-2.3643E-12*T+3.8504E-10)*T-9.7729E-9
      C2 = (((1.0405E-12*T-2.5335E-10)*T+2.5974E-8)*T-1.7107E-6)*T
     X     +3.1260E-5
      C1 = (((-6.1185E-10*T+1.3621E-7)*T-8.1788E-6)*T+6.8982E-4)*T
     X     +0.153563
      C0 = ((((3.1464E-9*T-1.47800E-6)*T+3.3420E-4)*T-5.80852E-2)*T
     X     +5.03711)*T+1402.388
      C = ((C3*P+C2)*P+C1)*P+C0
C SOUND SPEED RETURN
      SVEL = C + (A+B*SR+D*S)*S
      RETURN
      END
```

SOUND SPEED IN SEAWATER U [m/s]

PRESSURE DECIBARS	TEMPERATURE °C IPTS-68 0	10	20	30	40	SALINITY: 25
0	1435.8	1477.7	1510.3	1535.2	1553.4	
1000	1452.0	1494.1	1527.0	1552.1	1570.6	
2000	1468.6	1510.7	1543.6	1569.0	1587.6	
3000	1485.6	1527.5	1560.3	1585.7	1604.5	
4000	1502.8	1544.3	1576.9	1602.4	1621.3	
5000	1520.4	1561.3	1593.6	1619.0	1638.0	
6000	1538.1	1578.4	1610.3	1635.5	1654.6	
7000	1556.0	1595.6	1626.9	1651.9	1671.0	
8000	1574.1	1612.8	1643.5	1668.2	1687.2	
9000	1592.2	1630.1	1660.2	1684.5	1703.3	
10000	1610.4	1647.4	1676.8	1700.6	1719.2	
0	1442.5	1483.7	1515.9	1540.4	1558.3	SALINITY: 30
1000	1458.8	1500.2	1532.5	1557.3	1575.4	
2000	1475.4	1516.8	1549.2	1574.1	1592.3	
3000	1492.4	1533.6	1565.8	1590.8	1609.2	
4000	1509.7	1550.4	1582.4	1607.4	1626.0	
5000	1527.2	1567.4	1599.1	1624.0	1642.7	
6000	1544.9	1584.4	1615.7	1640.4	1659.2	
7000	1562.7	1601.5	1632.3	1656.8	1675.6	
8000	1580.7	1618.7	1648.9	1673.1	1691.8	
9000	1598.8	1636.0	1665.5	1689.3	1707.8	
10000	1616.8	1653.3	1682.1	1705.4	1723.5	
0	1449.1	1489.8	1521.5	1545.6	1563.2	SALINITY: 35
1000	1465.5	1506.3	1538.1	1562.4	1580.2	
2000	1482.3	1523.0	1554.7	1579.2	1597.1	
3000	1499.3	1539.7	1571.3	1595.9	1613.9	
4000	1516.5	1556.5	1587.9	1612.5	1630.7	
5000	1534.0	1573.4	1604.5	1629.0	1647.3	
6000	1551.6	1590.4	1621.0	1645.4	1663.8	
7000	1569.4	1607.5	1637.6	1661.7	1680.1	
8000	1587.2	1624.6	1654.1	1677.9	1696.2	
9000	1605.2	1641.8	1670.6	1694.0	1712.2	
10000	1623.2	1659.0	1687.2	1710.1	1727.8	
0	1455.8	1495.9	1527.1	1550.8	1568.1	SALINITY: 40
1000	1472.3	1512.5	1543.7	1567.6	1585.0	
2000	1489.1	1529.1	1560.3	1584.3	1601.8	
3000	1506.1	1545.8	1576.8	1600.9	1618.6	
4000	1523.3	1562.5	1593.3	1617.5	1635.3	
5000	1540.7	1579.4	1609.8	1633.9	1651.9	
6000	1558.2	1596.3	1626.3	1650.2	1668.3	
7000	1575.9	1613.3	1642.8	1666.5	1684.6	
8000	1593.7	1630.3	1659.2	1682.6	1700.6	
9000	1611.5	1647.4	1675.7	1698.7	1716.4	
10000	1629.3	1664.6	1692.2	1714.6	1732.0	

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